

# Model Question of HSC Examination 2020

Higher Mathematics 1<sup>st</sup> Paper (Creative) Subject Code : 

2	6	5
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Time — 2 hours 35 minutes

Full marks — 50

*[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]*

## Group A – Algebra & Geometry

1. ► Scenario-1 :  $\vec{P} = 2\hat{i} + \hat{j} + \hat{k}$  and  $\vec{Q} = \hat{i} - 2\hat{j} + \hat{k}$

Scenario-2 :  $M = \mathbb{R} - \left(-\frac{1}{2}\right)$ ,  $N = \mathbb{R} - \left(\frac{1}{2}\right)$  and  $g : M \rightarrow N$  is

defined by  $g(x) = \frac{x-3}{2x+1}$

a. Find the domain of  $f(x) = \sqrt{25-x^2}$ . 2

b. From scenario-1, find the unit vector which is perpendicular to the vector  $\vec{P}$  and  $\vec{Q}$ . 4

c. From scenario-2, show that,  $g(x)$  is one-one and onto. Also find  $g^{-1}(x)$  and  $g^{-1}\left(\frac{1}{3}\right)$ . 4

2. ★ Mr. 'X' is a professor who has drawn eight straight lines of length 2, 4, 6, 8, 10, 12, 14 and 16 cm respectively in the board.

a. If  $4 \times {}^n P_3 = 5 \times n - 1 P_3$ , then find the value of  $n$ . 2

b. How many permutations and combinations can be made from the letters of the underlined word in the stem. 4

c. Find the number of ways in which four straight lines can be chosen from the stem to form a quadrilateral. 4

3. ► Scenario-1 :  $6x - y = 1$ ,  $2x - 5y = 5$

Scenario-2 :  $2x - y = 3$

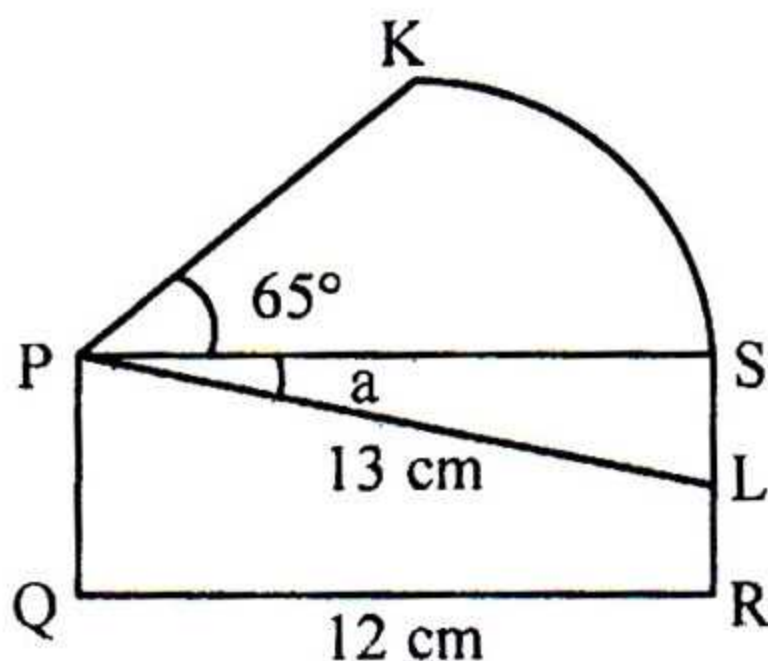
- a. For which value of  $m$  the matrix  $\begin{bmatrix} m-1 & 3 \\ 1 & -1 \end{bmatrix}$  is singular? 2
- b. From scenario-1, find the equation PQ if  $P(m, n)$  is a point on the 1<sup>st</sup> straight line and  $Q(n, m)$  is a point on the 2<sup>nd</sup> straight line. 4
- c. The centre of a circle lies on the straight line in scenario-2 and it passed through the points  $(3, -2)$  and  $(-2, 0)$ . Find the equation. 4
4. ► Cadet 'Y' writes a circle equation  $x^2 + y^2 - 2bx = 0$  and

matrix  $\begin{bmatrix} 3 & -4 & 2 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$  in the black board.

- a. Find the equation of the tangent at the point  $(4, -11)$  of the circle  $x^2 + y^2 - 3x + 10y - 15 = 0$  2
- b. Show that  $px + qy = 1$  will touch the circle in the stem if  $b^2q^2 + bpq = 1$ . 4
- c. Find the inverse matrix of the given matrix in the stem. 4

### Group B – Trigonometry & Calculus

5. ►



Here, PQRS is a rectangle and PKS is a circular segment.

- a. Show that,  $\sin 54^\circ + \cos 54^\circ = \sqrt{2} \cos 9^\circ$ . 2
- b. Find the area of the region bounded by PKSL. 4
- c. Draw the graph of the trigonometric ratio of the angle  $a$  which is denoted by  $\frac{PS}{PL}$ , where  $-2\pi \leq a \leq 2\pi$  4

6. **★** Scenario-1 :  $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$

Scenario-2 :  $x^2 + y^2 = 16$

- a. Find the value of  $\cos 38^\circ 15' \sin 68^\circ 15' - \cos 51^\circ 45' \sin 21^\circ 45'$ . 2
- b. Using the information in scenario-1, show that  $C = 45^\circ$  or  $135^\circ$ . 4
- c. With the help of integration, find the area bounded by the circle in scenario-2. 4

7. **★**  $\varphi(x) = 4e^x + 9e^{-x}$  and  $\theta = \frac{2\pi}{15}$

- a. Find the limit of  $\lim_{x \rightarrow a} \frac{x^{7/2} - a^{7/2}}{\sqrt{x} - \sqrt{a}}$ . 2
- b. Show that, minimum value of  $\varphi(x)$  is  $e$ . 4
- c. Using the value of  $\theta$  in the stem, show that,  $16\cos\theta\cos2\theta\cos4\theta\cos7\theta = 1$ . 4

8. **►** Scenario-1 :  $f(x) = \sin x$

Scenario-2 : i)  $\varphi(x) = \sqrt{16 - x^2}$ , ii)  $\omega(x) = \frac{\ln \sec^{-1} x}{x\sqrt{x^2 - 1}}$

- a. Find the derivative of  $x^y = y^x$  with respect to  $x$ . 2
- b. From scenario-1, find the derivative of  $f(mx)$  with respect to  $x$  by the first principal. 4
- c. From scenario-2, find i)  $\int_0^4 \varphi(x) dx$  ii)  $\int \omega(x) dx$  4

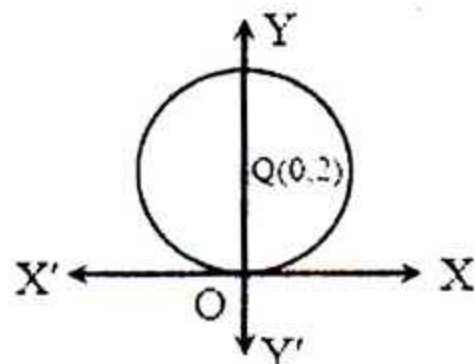
Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. If  $k = \begin{bmatrix} 4 & 5 \\ -1 & 6 \end{bmatrix}$ , then  $k-3I=?$
- (a)  $\begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix}$                       (b)  $\begin{bmatrix} 12 & 0 \\ 0 & 18 \end{bmatrix}$
- (c)  $\begin{bmatrix} 1 & 5 \\ -1 & 3 \end{bmatrix}$                       (d)  $\begin{bmatrix} -1 & 3 \\ 1 & 5 \end{bmatrix}$
2. What is the number of arrangements of the letters of the word "Multiplication" taking all at a time?
- (a)  $\frac{14!}{2!2!}$                       (b)  $\frac{14!}{2!3!}$
- (c)  $\frac{14!}{2!2!3!}$                       (d)  $\frac{14!}{2!3!3!}$
3. The equation of a straight line which is parallel to x-axis and which passes through the point of  $2x + y = 6$  and  $2x - y = -2$  is —
- (a)  $x - 1 = 0$                       (b)  $x + 1 = 0$
- (c)  $y + 4 = 0$                       (d)  $y - 4 = 0$
4. The circle  $x^2 + y^2 = 169$  has—
- i. Central (0, 0)  
ii. radius 13 unit  
iii. length of y-intercept is 26 unit
- Which one is correct?
- (a) i & ii                      (b) i & iii
- (c) ii & iii                      (d) i, ii & iii
5. What is the domain of  $g(x) = \frac{1}{\sqrt{13-x}}$ ?
- (a)  $(-\infty, 13]$                       (b)  $(-\infty, 13)$
- (c)  $[13, -\infty)$                       (d)  $(13, \infty)$
6. If  $\cos\theta = x$ , then—
- i.  $\cos 2\theta = 2x^2 - 1$   
ii.  $\sin 2\theta = 2x\sqrt{1-x^2}$   
iii.  $\tan\theta = \frac{2x\sqrt{1-x^2}}{1-x^2}$
- Which one is correct?
- (a) i & ii                      (b) i & iii
- (c) ii & iii                      (d) i, ii & iii
7. The value of  $2\cos^2 15^\circ$  is equal to—
- (a)  $\frac{2-\sqrt{3}}{2}$                       (b)  $\frac{2+\sqrt{3}}{2}$
- (c)  $\frac{1+\sqrt{3}}{2}$                       (d)  $\frac{-1+\sqrt{3}}{2}$
8. ★ Which one of the following is correct?
- (a)  $\tan(-\theta) = \tan\theta$
- (b)  $\sec(-\theta) = \sec\theta$
- (c)  $\operatorname{cosec}(-\theta) = \operatorname{cosec}\theta$
- (d)  $\cot(-\theta) = \cot\theta$
9. ★  $\int a^x dx = ?$  where  $a > 0, a \neq 1$
- (a)  $a^x \ln a$                       (b)  $\frac{a^x}{\ln a}$
- (c)  $a^x$                       (d)  $x \ln a$
10. Which one is the (2, 3)<sup>th</sup> cofactor of the determinant  $\begin{vmatrix} -1 & 5 & 7 \\ 8 & -9 & 2 \\ 1 & -4 & 1 \end{vmatrix}$ ?
- (a) -8                      (b) -1
- (c) 1                      (d) 8
11. How many ways can you select 4 books from 8 books?
- (a) 35                      (b) 70
- (c) 840                      (d) 1680
12. What is the slope of  $2x + 3y = 1$ ?
- (a) -2                      (b) -3
- (c)  $-\frac{3}{2}$                       (d)  $-\frac{2}{3}$

Answer the questions 13 & 14 based on the following stem :



The centre of the above circle is at C(0, 2).

13. Which one is the equation of the above circle in the figure?

- (a)  $x^2 + y^2 + 4y = 0$  (b)  $x^2 + y^2 - 4y = 0$   
 (c)  $x^2 + y^2 + 4x = 0$  (d)  $x^2 + y^2 - 4x = 0$

14. Which one is the equation of the tangent of the circle that is parallel to x-axis in the above figure?

- (a)  $x - 4 = 0$  (b)  $x + 4 = 0$   
 (c)  $y - 4 = 0$  (d)  $y + 4 = 0$

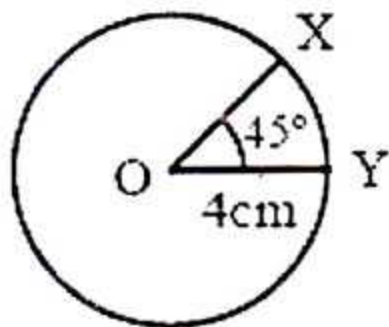
15.  $\hat{j} \times \hat{k} = ?$

- (a) 0 (b) 1  
 (c) 1 (d) -1

16. What is the period of  $\sin x$ ?

- (a)  $\frac{\pi}{2}$  (b)  $\pi$   
 (c)  $3\frac{\pi}{2}$  (d)  $2\pi$

Observe the following figure :



17. What is the area of the sector XOY in the above figure?

- (a)  $\frac{\pi}{4}$  (b)  $3\frac{\pi}{4}$   
 (c)  $\pi$  (d)  $2\pi$

18. ★ What is the value of  $\lim_{x \rightarrow \infty} \frac{5^x - 5^{-x}}{5x + 5^{-x}}$ ?

- (a) 0 (b) -1  
 (c) 1 (d)  $\infty$

Answer the following questions 19 and 20 based on the following stem :

$$P = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, Q = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

19. Which one is the order of PQ?

- (a)  $1 \times 2$  (b)  $2 \times 1$   
 (c)  $1 \times 1$  (d)  $2 \times 2$

20. What is the value of  $\text{adj}(P)$ ?

- (a)  $\begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$  (b)  $\begin{pmatrix} -4 & 3 \\ 1 & -2 \end{pmatrix}$   
 (c)  $\begin{pmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{pmatrix}$  (d)  $\begin{pmatrix} -4/5 & 3/5 \\ 1/5 & -2/5 \end{pmatrix}$

21. ★ All the letters of the word 'CADET' can be—

- i. arranged in 51 ways  
 ii. rearranged in 121 ways  
 iii. selected in 1 way

Which one is correct?

- (a) i & ii (b) i & iii  
 (c) ii & iii (d) i, ii & iii

22. What is the Cartesian co-ordinates of the point  $(-2, 120^\circ)$ ?

- (a)  $(1, -\sqrt{3})$  (b)  $(\sqrt{3}, -1)$   
 (c)  $(-1, \sqrt{3})$  (d)  $(\sqrt{3}, 1)$

23. The centre of the circle  $x^2 + y^2 - 8x - 6y + 16 = 0$  is—

- (a)  $(-4, -3)$  (b)  $(4, 3)$   
 (c)  $(-8, -6)$  (d)  $(8, 6)$

24. If  $f(x) = 3x$ , then—

i.  $\frac{d}{dx}[\sec\{f(x)\}] = 3\tan^2 3x$

ii.  $\int_0^{\pi/2} \sin\{f(x)\} dx = \frac{1}{3}$

iii.  $\int \frac{1}{f(x)} dx = \frac{1}{3} \ln x + C$

Which one is correct?

- (a) i & ii (b) i & iii  
 (c) ii & iii (d) i, ii & iii

25. How many ways can 8 cadets fall in the circular form?

- (a) 2520 (b) 5040  
 (c) 20160 (d) 40320

Ans.	1	(c)	2	(c)	3	(d)	4	(d)	5	(a)	6	(a)	7	(b)	8	(b)	9	(b)	10	(c)	11	(b)	12	(d)	13	(b)
	14	(c)	15	(a)	16	(d)	17	(d)	18	(c)	19	(b)	20	(a)	21	(b)	22	(a)	23	(b)	24	(c)	25	(b)		