

Model Question of HSC Examination 2020

Higher Mathematics 1st Paper (Creative) Subject Code :

2	6	5
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Time — 2 hours 35 minutes

Full marks — 50

[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]

Group A – Algebra & Geometry

1. ★ Given, $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$ and $D =$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & p & p^2 \\ 1 & p^2 & p^4 \end{vmatrix}$$

- a. Find the component of vector \vec{B} along \vec{A} . 2
- b. Prove that, $D = p(p-1)^2(p^2-1)$. 4
- c. Determine the orthogonal unit vector on the plane that is formed by two vectors \vec{A} and \vec{B} . 4

2. ► $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$ are two functions.

$3x - y + 7 = 0$ is the equation of straight line.

- a. Find fog and fof. 2
- b. Find the distance of (1,2) from $3x + y + 4 = 0$ measured parallel to $3x - y + 7 = 0$ 4

c. Find the equation of straight lines passing through $(-1,2)$ and makes an angle 45° with the line $3x - y + 7 = 0$ and show that, they are perpendicular to each other. 4

3. ► The circle $x^2 + y^2 - 4x - 6y + c = 0$ touches the $x -$ axis.

a. Find radius and the length of the intercept which cuts off from the $x -$ axis. 2

b. Find c and the co-ordinates of the point of contact. 4

c. Find the equation of the circle which is concentric with the given circle and touches the $x -$ axis : and also find the length of the intercept it makes on the $y -$ axis. 4

4. ► The live interaction with the crew can help students envision career possibilities resulting from studying Science, Technology, Engineering and Mathematics.

a. Show that, ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$. 2

b. In how many of them the three E's stand together and how many will they stand first of the word 'Engineering'. 4

c. Find how many arrangements can be made with the letters of the word 'MATHEMATICS' so that the vowels do not

change their order and in how many of them the vowels occur together? 4

Group B – Trigonometry & Calculus

5. ★ $\tan \alpha + \sec \alpha = p$ and, $f(x) = \sin x$.

a. If $\operatorname{cosec} A + \operatorname{cosec} B + \operatorname{cosec} C = 0$ prove that, $(\sum \sin A)^2 = \sum \sin^2 A$ 2

b. Show that, $\sin \alpha = \frac{p^2 - 1}{p^2 + 1}$ 4

c. Solve graphically $f(2x)$ within the limit $0^\circ \leq x \leq 180^\circ$. 4

6. ► $y^2 = 4 + 3 \sin x$.

a. Find the derivative of $\frac{x \sin x}{1 + \cos x}$ with respect to x . 2

b. For which values of x , $\frac{y^2}{3}$ is maximum or minimum? Find the values. 4

c. Show that, $2y \frac{d^2 y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 + y^2 = 4$. 4

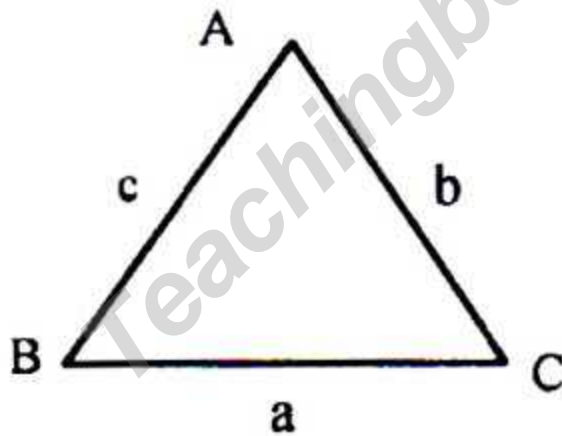
7. $x^2 + y^2 = 16$ is a circle equation and $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is an ellipse equation.

a. integrate : $\int e^x (\sin x + \cos x) dx$.

b. From given circle equation find the value of $\int y dx$. 4

c. Find the area of the given ellipse. 4

8. ►



a. Prove that, $\tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ$ 2

b. If $A + B + C = \frac{\pi}{2}$, show that, $\cos^2 A + \cos^2 B - \cos^2 C = 2$

$\cos A \cos B \sin C$. 4

c. For $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, find the angle of C. 4

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. **★** Which one is the integration of $\int \ln x dx$?

- (a) $x \ln x$
 (b) $x \ln x - x$
 (c) $x \ln x + c$
 (d) $x \ln x - x + c$

2. What is the value of $\lim_{x \rightarrow 0} (1 + 5x)^{\frac{3x+2}{x}}$?

- (a) 1 (b) ∞
 (c) e^{10} (d) 6

3. You have 6 friends. In how many ways you can invite one or more of them to a dinner?

- (a) 63 (b) 64
 (c) 32 (d) 31

4. If any chord of $x^2 + y^2 = 81$ is bisected at $(-2, 3)$, what is the equation of that chord?

- (a) $2x - 3y + 13 = 0$
 (b) $2x + 3y - 13 = 0$
 (c) $2x + 3y - 6 = 0$
 (d) $3x + 2y - 13 = 0$

5. In $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

- i. (1, 1)-th cofactor is $b_2c_3 - b_3c_2$.
 ii. (1, 2)-th cofactor is $-a_2c_3 + a_3c_2$.
 iii. (2, 3)-th cofactor is $-a_1b_3 + a_3b_1$.

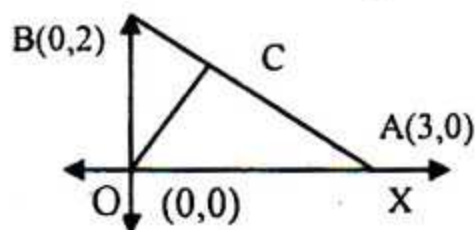
Which one of the following is correct?

- (a) i & ii (b) ii & iii
 (c) i & iii (d) i, ii & iii

6. What is the derivative of e^{4x} ?

- (a) 0 (b) $ax \ln a$
 (c) ae^{ax} (d) ae^a

In figure $OC \perp AB$ answer the questions 7 and 8 :



7. What is the equation of OC?

- (a) $2x + 3y - 6 = 0$ (b) $3x - 2y = 0$
 (c) $3x + 2y = 0$ (d) $3x - 2y + 6 = 0$

8. According to above stem –

- i. Area of $\triangle OAB$ is 3 square units.
 ii. Slope of OC is $-\frac{3}{2}$.

iii. Polar co-ordinates of B is $(2, 90^\circ)$.

Which one of the following is correct?

- (a) i & ii (b) ii & iii
 (c) i & iii (d) i, ii & iii

9. Length of three sides of a triangle are 3, 5 and 7. Which is the value of its obtuse angle?

- (a) 30° (b) 60°
 (c) 120° (d) 180°

10. What is the value of nC_n ?

- (a) 0 (b) $0!$
 (c) 1 (d) $n!$

11. If $A = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$, What is A^T ?

- (a) $\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$
 (c) $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$ (d) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

12. What is the derivative of $\frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}$

with respect to x?

- (a) $\sin x - \cos x$
 (b) $\sin x + \cos x$
 (c) 1
 (d) 0

13. **★** Find the distance between the parallel straight lines $4x - 3y + 2 = 0$ and $8x - 6y - 9 = 0$.

- (a) $\frac{10}{13}$ (b) $\frac{13}{10}$
 (c) $\frac{13}{2}$ (d) $\frac{9}{2}$

14. Which one is the derivative of $\sin^{-1}(2x\sqrt{1-x^2})$?

- (a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{2x}{\sqrt{1-x^2}}$
 (c) $-2x\frac{1}{\sqrt{1-x^2}}$ (d) $2\frac{1}{\sqrt{1-x^2}}$

15. What is the length of intercept made by $x^2 + y^2 + 4x + 6y - 12 = 0$ on the y-axis?

- (a) 8 (b) 21
 (c) $\sqrt{16}$ (d) $2\sqrt{21}$

16. What is the angle between $2\hat{i} - \hat{j} + 2\hat{k}$ and x-axis?

- (a) $\cos^{-1}\left(\frac{1}{2}\right)$
 (b) $\cos^{-1}\left(\frac{2}{3}\right)$
 (c) $\cos^{-1}\left(\frac{-1}{3}\right)$
 (d) $\cos^{-1}\left(\frac{2}{3}\right)$

17. Find the equation of straight line which passes through (4,-3) and parallel to $2x + 11y - 2 = 0$.

- (a) $2x + 11y - 25 = 0$
 (b) $11x + 2y - 25 = 0$
 (c) $11x - 2y - 25 = 0$
 (d) $2x + 11y + 25 = 0$

18. $x^2 + y^2 - 6x + 8y + 9 = 0$ whose —

- i. center is (3, -4).
 ii. radius is 4.
 iii. length of tangent from the point (1, -1) is 25.

Which one of the following is correct?

- (a) i, ii (b) ii, iii
 (c) i, iii (d) i, ii & iii

19. What is the value of $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$?

- (a) 1 (b) 2
 (c) 3 (d) 4

20. Which one is the integration of

$$\int \frac{\tan x}{\ln \cos x} dx?$$

- (a) $\ln(\ln \cos x) + c$
 (b) $-\ln \cos x + c$
 (c) $-\ln(\ln \cos x) + c$
 (d) $(\ln \cos x) + c$

Answer 21 & 22 based on the following data:

The line $3x + 4y = k$ touches circle $x^2 + y^2 = 10x$.

21. What is the centre of given circle?

- (a) (0, 0) (b) (0, -5)
 (c) (5, 5) (d) (5, 0)

22. What is the value of k?

- (a) 10 (b) 20
 (c) 30 (d) 40

23. The matrix $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a —

- i. diagonal matrix.
 ii. scalar matrix.
 iii. involutory matrix.

Which one of the following is correct?

- (a) i, ii
 (b) ii, iii
 (c) i, iii
 (d) i, ii & iii

24. ★ The equation of the circle that touches y-axis at point (0,4) and whose centre lies on the line $5x - 7y - 2 = 0$

- (a) $x^2 + y^2 + 12x - 8y + 16 = 0$
 (b) $x^2 + y^2 - 8x - 6y + 8 = 0$
 (c) $x^2 + y^2 - 12x - 8y + 16 = 0$
 (d) $x^2 + y^2 + 8x + 6y - 40 = 0$

25. ★ What is the value of $\cos 68^\circ 20' \cos 8^\circ 20' + \cos 81^\circ 40' \cos 21^\circ 40'$?

- (a) 1 (b) $\frac{\sqrt{3}}{2}$
 (c) $\frac{1}{2}$ (d) 0

Ans.	1	(d)	2	(c)	3	(a)	4	(a)	5	(d)	6	(c)	7	(b)	8	(c)	9	(c)	10	(b)	11	(c)	12	(d)	13	(b)
	14	(d)	15	(a)	16	(b)	17	(d)	18	(a)	19	(d)	20	(c)	21	(d)	22	(d)	23	(a)	24	(c)	25	(c)		