Model Question of HSC Examination 2020

Higher Mathematics 2nd Paper (Creative) Subject Code : 2 6 6

Time — 2 hours 35 minutes Full marks — 50

[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]

Group A – Algebra & Trigonometry

1. ▶ $z = px + qy$; Constraints: $x + y \le 7$, $2x + 5y \le 20$, $x \ge 0$, $y \ge 0$

a. Find the square root of $2i$. 2

b. If $p = 3$, $q = 4$ find the maximum value of z using graph. 4

c. Show that $4(x^2 - y^2) = \frac{a}{x} + \frac{b}{y}$, if $\sqrt[3]{a + ib} = z$ when $p = 1$ and $q = i$. 4

2. $x^2 + px + q = 0$..... (i), $cx^2 + bx + a = 0$..... (ii) and $ax^2 + bx + c = 0$..... (iii)

a. Show that, the roots of the equation $x^2 - (a + b)x + \frac{1}{2}(a^2 + b^2) = 0$ cannot be real unless $a = b$. 2

b. If the difference of the roots of (i) is 1, show that $p^2 + 4q^2 = (1 + 2q)^2$ 4

c. If one of the roots of (iii) is double one of the roots of (ii), show that, either $2a - c = 0$ or, $(2a + c)^2 - 2b^2 = 0$. 4

3. ▶ $A = (a + 3x)^n$, $B = (1 - 4x)^{-\frac{1}{2}}$

a. If $y = x + x^2 + x^3 + ...$ then show that $x = y - y^2 + y^3 - y^4 + ...$ 2

b. Show that, the co-efficient of x^r in the expansion of B is $\frac{(2r)!}{(r!)^2}$. 4

c. If in the expansion of 'A' 1st three terms are respectively b, $\frac{21}{2}$ bx and $\frac{189}{4}$ bx^2, find the value of a, b and n. 4

4. ▶ $f(x) = \cos x$

a. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$. 2

b. Solve: $f(\theta).f(2\theta).f(3\theta) = \frac{1}{4}$; $0 < \theta < \pi$ 4

c. If $f(\pi\sin\theta) = \sin\{\pi f(\theta)\}$, show that $\theta = \pm\frac{1}{2}\sin^{-1}\frac{3}{4}$. 4

Group B - Geometry, Mechanics and Probability

5. ▶ $2x + y = 1$..... (i) $S_1 \equiv (1, 1)$, $e = \sqrt{3}$, $\frac{x^2}{p} + \frac{y^2}{5^2} =$ 1.....(ii), $A \equiv (6, 4)$

a. The focal distance of a point on the parabola $y^2 = 16x$ is 6, find the co-ordinates of that point. 2

b. For what value of 'p' does (ii) passes through the point 'A'. Find the vertex, eccentricity and co-ordinates of the foci of (ii). 4

c. Find the equation of the hyperbola whose directrix is (i), focus S_1 and eccentricity be 'e'. 4

6. ✪ Scenery-I : Three forces P, Q, R acting at a point are in equilibrium and the angle between P and Q is double the angle between P and R.

Scenery-II : Two unlike parallel forces P, Q (P > Q) act at A and B respecttvely. If P and Q are both increased by R and distance is 'd'.

a. Write down Lamy's Theorem. 2

b. From Scenery-I, prove that $R^2 = Q(Q - P)$. 4

c. From Scenery-II, show that $d = \frac{R}{P - Q}$, AB 4

7. ▶ Scenery-I : A particle moving along a straight line with uniform acceleration describes successive equal distances in times t_1, t_2 and t_3.

Scenery-II : A stone falling from the top of a vertical tower has descend x metres when another is let fall from a point y metres below the top. They fall from rest and reach the ground together.

a. A projectile is projected with an initial velocity of 21 ms^{-1} at an angle 30° with the horizon. Find the greatest height. 2

b. From Scenery-I, Show that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$ 4

c. From Scenery-II, show that the height of the tower is $\frac{(x + y)^2}{4x}$ metres. 4

8. ✪ Scenery-I : A, B are independent and $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$

Scenery-II : Two dice are thrown simultaneously.

a. Define sample space. 2

b. From scenery-I, find $P(A \cup B)$ and $P(A \cap B)$. 4

c. From scenery-II, write down the sample space and find the probability of appearing two sixes. 4

Higher Mathematics 2nd Paper (MCQ)

Subject Code : | 2 | 6 | 6 |

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. Which one is the inf of S = $\left\{(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots\ldots\ldots)\right\}$?

ⓐ $\frac{1}{2}$ ⓑ 0

ⓒ 1 ⓓ ∞

2. The largest or smallest value of the objective function is called —

ⓐ optimal value ⓑ optimal solution

ⓒ feasible region

ⓓ decision variables

3. What is the solution of $|4x - 2| \leq 18$?

ⓐ $-4 < x < 5$ ⓑ $-4 < x \leq 5$

ⓒ $-4 \leq x < 5$ ⓓ $-4 \leq x \leq 5$

4. If p, q, r ∈ R, then

i. $p + q \in R, pq \in R$

ii. $(p + q) + r = p + (q + r)$

iii. $p(q + r) = pq + pr$

Which one is correct?

ⓐ i, ii ⓑ ii, iii

ⓒ i, iii ⓓ i, ii, iii

α, β be the roots of the equation $2x^2 - 10x + 15 = 0$

From the above information, answer the questions no. 5-6 :

5. The discriminant of the above equation is —

ⓐ natural ⓑ complex

ⓒ real ⓓ fraction

6. $\sum a^3 = ?$

ⓐ $\frac{25}{2}$ ⓑ 125

ⓒ $\frac{475}{2}$ ⓓ 475

7. Which one is the argument of z = 2 − 5i?

ⓐ $-\tan^{-1}\frac{5}{2}$ ⓑ $\tan^{-1}\frac{5}{2}$

ⓒ $\pi - \tan^{-1}\frac{5}{2}$ ⓓ $-\pi + \tan^{-1}\frac{5}{2}$

8. What is the co-efficient of x^8 in $\frac{1-x}{1+x}$?

ⓐ − 1 ⓑ 0

ⓒ 1 ⓓ 2

9. The ellipse $\frac{x^2}{25} + \frac{y^2}{49} = 1$ has —

i. eccentricity $\frac{2\sqrt{6}}{7}$

ii. length of latus rectum $\frac{50}{7}$ units

iii. foci $(\pm 1, 0)$

Which one is correct?

ⓐ i, iii ⓑ i, ii

ⓒ ii, iii ⓓ i, ii, iii

10. General solution of the equation $\sqrt{3}\cos\theta + \sin\theta = 2$ is —

ⓐ $\theta = 2n\pi$ ⓑ $\theta = (12n - 1)\frac{\pi}{6}$

ⓒ $\theta = (12n + 1)\frac{\pi}{6}$ ⓓ $\theta = (4n - 1)\frac{\pi}{6}$

11. A pack of 52 cards contains 4 aces. One card is drawn a random. What is the probability of not getting an ace?

ⓐ $\frac{12}{13}$ ⓑ $\frac{4}{13}$

ⓒ $\frac{1}{13}$ ⓓ $\frac{2}{13}$

12. Two forces 3P and 5P act so that they are perpendicular to each other. What is their resultant?

ⓐ 3P ⓑ $2\sqrt{2}$ P

ⓒ $\sqrt{34}$ P ⓓ $\sqrt{43}$ P

13. If x + iy and x − iy are complex numbers, then their —

i. sum is real

ii. product is real

iii. difference is real

Which one is correct?

ⓐ i, ii ⓑ ii, iii

ⓒ i, iii ⓓ i, ii, iii

14. If $e > 1$, then what type of conic is formed .....
ⓐ circle ⓑ parabola
ⓒ ellipse ⓓ hyperbola

15. ★ What is the value of 'y' if $y = \sin\left(2\tan^{-1}\frac{2}{5}\right)$?
ⓐ $\frac{20}{29}$ ⓑ $\frac{20}{21}$
ⓒ $\frac{21}{20}$ ⓓ $\frac{29}{20}$

16. A particle is projected with velocity u at an angle α to the horizon then.....
i. horizontal range, $R = \frac{u^2\sin 2\alpha}{g}$
ii. time of rise $= \frac{u\sin\alpha}{g}$
iii. $R_{max} = \frac{u^2}{g}$
Which one is correct?
ⓐ i, ii ⓑ ii, iii
ⓒ i, iii ⓓ i, ii, iii

17. $\frac{5x}{(2-3x)(1-x)} = ?$
ⓐ $\frac{-5}{1-x} - \frac{10}{2-3x}$ ⓑ $\frac{5}{x-1} + \frac{10}{2-3x}$
ⓒ $\frac{-5}{1-x} + \frac{10}{2-3x}$ ⓓ $\frac{5}{1-x} + \frac{10}{2-3x}$

18. Solution of $\tan\theta + \cot\theta = 2$ is $\left(0° < \theta < \frac{\pi}{2}\right)$
ⓐ $0°$ ⓑ $\frac{\pi}{6}$
ⓒ $\frac{\pi}{4}$ ⓓ $\frac{\pi}{3}$

A particle starting with velocity u moves for 5 sec. With constant acceleration f describes 90 metres the acceleration then ceases and describes 80 metres during the next 5 sec. From the above information, answer the question no. 19-20

19. What is the final velocity (v)?
ⓐ $16ms^{-1}$ ⓑ $17ms^{-1}$
ⓒ $18ms^{-1}$ ⓓ $20ms^{-1}$

20. What is the value of 'u'?
ⓐ $16ms^{-1}$ ⓑ $17ms^{-1}$
ⓒ $18ms^{-1}$ ⓓ $20ms^{-1}$

21. Which one is the range of $y = \tan^{-1} x$?
ⓐ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ⓑ $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right]$
ⓒ $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ ⓓ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right[$

22. ★ If P, Q, R make angles α, β, θ respectively with OX then .....
i. $R\cos\theta = P\cos\alpha + Q\cos\beta$
ii. $R\sin\theta = P\sin\alpha + Q\sin\beta$
iii. $R = P + Q$
Which one is correct?
ⓐ i ⓑ ii
ⓒ iii ⓓ i, ii, iii

The parabola $y^2 = 4Px$ passes through the points (3, –2).
From the above information, answer the questions no. 23-24 :

23. What is the value of 'P'?
ⓐ $\frac{1}{3}$ ⓑ $\frac{3}{4}$
ⓒ 3 ⓓ 4

24. What is the co-ordinates of focus?
ⓐ $\left(-\frac{1}{3}, 0\right)$ ⓑ $\left(0, -\frac{1}{3}\right)$
ⓒ $\left(0, \frac{1}{3}\right)$ ⓓ $\left(\frac{1}{3}, 0\right)$

25. ★ If one root of the equation $x^2 - 8x - P = 0$ is 6 then P = ?
ⓐ – 12 ⓑ – 8
ⓒ 8 ⓓ 12

Ans.																														
	1	ⓑ	2	ⓐ	3	ⓓ	4	ⓓ	5	ⓑ	6	ⓐ	7	ⓐ	8	ⓓ	9	ⓑ	10	ⓒ	11	ⓐ	12	ⓒ	13	ⓐ	14	ⓓ	15	ⓐ
	16	ⓓ	17	ⓒ	18	ⓒ	19	ⓐ	20	ⓓ	21	ⓒ	22	ⓓ	23	ⓐ	24	ⓓ	25	ⓐ										