

Model Question of HSC Examination 2020

Higher Mathematics 2nd Paper (Creative) Subject Code :

2	6	6
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Full marks — 50

Time — 2 hours 35 minutes

[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]

Group A – Algebra & Trigonometry

1. ► $z_1 = a + ib$ and $z_2 = c + id$, where a, b, c and d are real numbers.

a. Why is every real number it self a complex number? 2

b. If $a = -8$ and $b = -6$, find the square root of z_1 . 4

c. If $x:y = z_1 : z_2$, show that $(c^2 + d^2)x^2 - 2(ac + bd)xy + (a^2 + b^2)y^2 = 0$. 4

2. ► Consider $A^m = (a + bx)^m$; $a, b, m \in \mathbb{R}$.

a. If $y = x + x^2 + x^3 + \dots$, then express x in terms of y . 2

b. If $a = b = 1$ and if the 21st and 22nd terms in the expansion of A_{44} are equal, find the value of x . 4

c. If $a = 1$ and $b = -4$, show that the coefficient of x^r in the

expansion of $A^{-\frac{1}{2}}$ is $\frac{(2r)!}{(r!)^2}$. 4

3. **★** α and β are the roots of the equation $px^2 + qx + r = 0$.
- a. Express the roots of the equation $rx^2 - 2qx + 4p = 0$ in terms of α and β . 2
- b. Prove that $(p\alpha + q)^{-2} + (p\beta + q)^{-2} = \frac{q^2 - 2pr}{p^2 r^2}$. 4
- c. If α is double of one of the roots of $rx^2 + qx + p = 0$, then show that either $2p = r$ or $(2p + r)^2 = 2q^2$. 4
4. **►** Consider the function $f(x) = \tan x$.
- a. Is $\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$, $-1 \leq x \leq 1$ true? Justify your contention. 2
- b. If $\sin^{-1} \frac{2f(a)}{1 + f^2(a)} - \cos^{-1} \frac{1 - f^2(b)}{1 + f^2(b)} = 2 \tan^{-1} x$, show that $x = \frac{f(a) - f(b)}{1 + f(a)f(b)}$ 4
- c. Solve: $f(x) + \frac{1}{f(x)} = 2\sqrt{1 + f^2(x)}$, $-2\pi < x < 2\pi$. 4


Group B - Geometry, Mechanics and Statistics

5. **★** The foci of a hyperbola are $(8, 4)$ and $(16, 4)$ and its eccentricity is 4.

a. For what value of p does the parabola $y^2 = 4px$ pass through the points $(3, \pm 2)$? Find, also, the length of its latus rectum for this value of p . 2

b. Find the equation of an ellipse with eccentricity $\frac{1}{\sqrt{2}}$ which is concentric with the stimulus hyperbola and passes through the stimulus points. 4

c. Find the equation of the hyperbola. 4

6.  The resultant of two forces P and Q ($P > Q$) acting at a point O is R and the angle between them is α .


a. State the parallelogram law of forces. 2

b. If $F = R_{\max}$ and $G = R_{\min}$, then show that $R =$

$$\sqrt{F^2 \cos^2 \frac{\alpha}{2} + G^2 \sin^2 \frac{\alpha}{2}}. \quad 4$$

c. If a transversal intersects the lines of action of the three stimulus forces P , Q and R at the points L , M and N , then

show that $\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$. 4

7.  Considering the point of projection as origin O and OX , OY as x and y -axes respectively, a particle projected in vacuo from O with a velocity u at an angle α to the horizon reaches at

the point $P(x, y)$ at a certain time and finally falls on the projection plane at a distance R .

a. Find the value of R . 2

b. Show that $y = x \tan \alpha \left(1 - \frac{x}{R}\right)$. 4

c. If the particle passes through the points $Q(s, b)$ and $R(b, a)$, then prove that $R = \frac{a^2 + ab + b^2}{a+b}$. 4

8. ► The probabilities of X and Y for solving a sum are $\frac{1}{3}$ and $\frac{1}{4}$ respectively and the probability of their solving it together is $\frac{1}{2}$. They, also, form a set S taking all the multiples of 4 not greater than 48.

a. If the mean of the marks obtained by each of the students A and B is 84 and their standard deviations of marks are respectively 12.12 and 4.04, whose marks are more spread out? 2

b. Ascertain whether X and Y are independent. If possible, show that $P(X/Y) = P(x)$. 4

c. Find the variance of the elements of S . 4

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. **★** What is the square root of $2i$?

- (a) $\pm\sqrt{1-i}$ (b) $\pm(1-i)$
 (c) $\pm\sqrt{1+i}$ (d) $\pm(1+i)$

2. If $a, b \in \mathbb{R}$, then which one of the following is correct?

- (a) $a < 0, b < 0 \Rightarrow ab < 0$
 (b) $a < 0, b < 0 \Rightarrow ab > 0$
 (c) $a < 0, b > 1 \Rightarrow ab > a$
 (d) $a > 0, b > 0 \Rightarrow ab < 0$

3. If $\sin^{-1} x + \cos^{-1} y = \frac{\pi}{2}$, then —

i. $x = y = \frac{1}{\sqrt{2}}$

ii. $x^2 + y^2 = 1$

iii. $x\sqrt{1-y^2} + y\sqrt{1-x^2} = 1$

Which one is correct?

- (a) i and ii (b) ii and iii
 (c) i and iii (d) i, ii and iii

4. For the hyperbola $16x^2 - 25y^2 = 400$, —

- i. Co-ordinates of vertices are $(\pm 5, 0)$
 ii. Equations of asymptotes are $y = \pm \frac{4}{5}x$
 iii. Length of latus rectum is $\frac{32}{25}$.

Which one is correct?

- (a) i and ii (b) ii and iii
 (c) i and iii (d) i, ii and iii

5. What is the value of the middle term of

the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^4$?

- (a) -2 (b) 20
 (c) 28 (d) 70

Consider the quadratic equation $4x^2 - 8(p-2)x + 1 = 0$. Answer 6-7:

6. If the roots of the stimulus equation are real and equal, then one of the values of p is —

- (a) $\frac{2}{5}$ (b) $\frac{2}{3}$
 (c) $\frac{4}{3}$ (d) $\frac{5}{2}$

7. If the roots of the stimulus equation are irrational and unequal, then which one of the following is one of the values of P ?

- (a) 2 (b) $\frac{7}{4}$
 (c) $\frac{9}{4}$ (d) 3

8. Which one of the following is one of the values of,

$$\sqrt{-3 + 3\sqrt{-3 + 3\sqrt{-3 + \dots \dots \infty}}}$$

(a) $\frac{3 + \sqrt{3}i}{2}$ (b) $\frac{2 + \sqrt{3}i}{2}$
 (c) $\frac{2 - \sqrt{3}i}{2}$ (d) $\frac{-3 - \sqrt{3}i}{2}$

9. What is the value of $\left(\frac{-1 + \sqrt{3}i}{2}\right)^{16} + \left(\frac{-1 - \sqrt{3}i}{2}\right)^{19}$?

- (a) -1 (b) 0
 (c) 1 (d) 2

10. **★** If a small rigid body is thrown vertically upward with velocity u , what is the time of its flight?

- (a) $\frac{u}{g}$ (b) $\frac{2u}{g}$
 (c) $\frac{u^2}{g}$ (d) $\frac{2u^2}{g}$

Two forces P and Q act a point inclined at a certain angle. Answer 11 - 12:

11. What is the sum of their maximum and minimum resultants?

- (a) $\sqrt{2(P^2 + Q^2)}$ (b) $\sqrt{2(P + Q)}$
 (c) $2Q$ (d) $2P$

12. If P and Q are each equal to their resultant, then what is the angle between P and Q ?

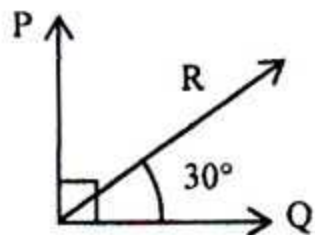
- (a) 0° (b) 60°
 (c) 90° (d) 120°

13. What is the value of ${}^{100}C_2 + {}^{100}C_3 + {}^{100}C_4 + {}^{100}C_{99} + {}^{100}C_{100}$?

- (a) $2^{100} - 101$ (b) $2^{100} - 1$
 (c) 2^{100} (d) $2^{100} + 101$

14. In figure, -

- i. $P = \frac{1}{2} R$
- ii. $Q = \frac{\sqrt{3}}{2} R$
- iii. $Q = \frac{1}{\sqrt{3}} P$.



Which one is correct?

- (a) i and ii
 - (b) ii and iii
 - (c) i and iii
 - (d) i, ii and iii
15. Which one of the following is a tangent to the parabola $y^2 = 8x$?
- (a) $y = x + 2$
 - (b) $y = x + 1$
 - (c) $y = x - 1$
 - (d) $y = x - 2$

16. What is the solution of the inequality $x^2 - 5x - 6 \leq 0$?

- (a) $2 \leq x \leq 3$
- (b) $-1 \leq x \leq 6$
- (c) $x \geq -1$ or $x \geq 6$
- (d) $x \geq 2$ or $x \geq 3$

Consider the equation $\sin \theta = a$

Answer 17-18:

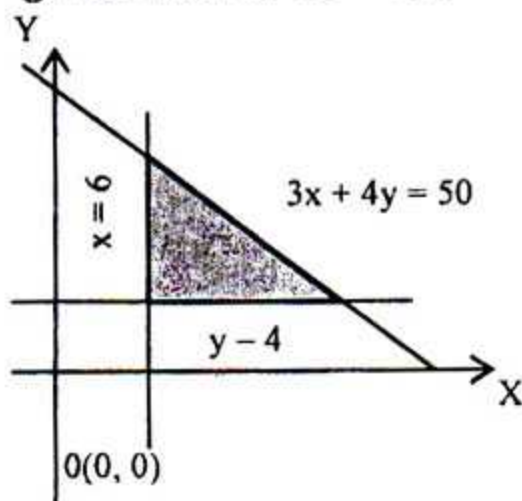
17. If $a = 1$, what is the value of θ ?

- (a) $\theta = 2n\pi + \frac{\pi}{2}$
- (b) $\theta = (2n + 1) \frac{\pi}{2}$
- (c) $\theta = (2n - 1) \frac{\pi}{2}$
- (d) $\theta = 2n\pi - \frac{\pi}{2}$

18. If $a = -1$, what is the value of θ ?

- (a) $\theta = (4n + 1) \frac{\pi}{2}$
- (b) $\theta = (2n + 1) \frac{\pi}{2}$
- (c) $\theta = (4n - 1) \frac{\pi}{2}$
- (d) $\theta = (2n - 1) \frac{\pi}{2}$

According to the information given answer 19 - 20.



19. Which one of the following represents the feasible region shown above in figure?

- (a) $3x + 4y > 50; x \geq 6, y \geq 4$
- (b) $3x + 4y \geq 50; x \geq 6, y \geq 4$
- (c) $3x + 4y \leq 50; x \geq 6, y \geq 4$
- (d) $3x + 4y < 50; x \geq 6, y \leq 4$

20. If the objective function is $z = x + y$ (x and y being positive integral values), what is the maximum value of z ?

- (a) 10
- (b) 14
- (c) 15
- (d) 19

21. Range of a missile on the earth is 1500m. If the acceleration due to gravity of moon is $\frac{1}{6}$ th of that of the earth, what is the range on the surface of the moon under the similar circumstances?

- (a) 250 m
- (b) 500 m
- (c) 1500m
- (d) 9000m

22. A bullet fired into a target loses half of its velocity after penetrating 4 inches. How far will it penetrate in inch?

- (a) 2
- (b) $\frac{4}{3}$
- (c) 1
- (d) $\frac{2}{3}$

23. If two dice and 3 coins are thrown together, what is the total number of sample points?

- (a) 288
- (b) 246
- (c) 72
- (d) 36

24. What is the standard deviation of the numbers - 27 and 33?

- (a) 10
- (b) 20
- (c) 30
- (d) 40

25. The variance of first n natural numbers is -

- (a) $\frac{n-1}{12}$
- (b) $\frac{n+1}{12}$
- (c) $\frac{n^2-1}{12}$
- (d) $\frac{n^2+1}{12}$

Ans.

1	(d)	2	(b)	3	(a)	4	(a)	5	(d)	6	(d)	7	(d)	8	(a)	9	(a)	10	(b)	11	(d)	12	(d)	13	(a)	14	(a)	15	(a)
16	(b)	17	(a)	18	(c)	19	(c)	20	(c)	21	(d)	22	(b)	23	(a)	24	(c)	25	(c)										