

Model Question of HSC Examination 2020

Higher Mathematics 2nd Paper (Creative) Subject Code :

2	6	6
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Full marks — 50

Time — 2 hours 35 minutes

[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]

Group A – Algebra & Trigonometry

1. ► Suppose $f(x) = 5x^2 - 16x + 3$, $g(x) = \frac{(2x - 3)(x - 2)^2}{x + 1}$ and

$h(x) = bx - c$ where $b, c > 0$.

a. Find the Sup S if $S = \{x \in \mathbb{R} : f(x) < 0\}$. 2

b. Solve the inequality $g(x) > 0$ then show the solution set on the number line. 4

c. If $|h(x)| < c$, then prove that $|x^3 + 1| < \frac{8c^3 + b^3}{b^3}$. 4

2. ► Suppose $p = \frac{-1 + \sqrt{-3}}{2}$, $q = \frac{-1 - \sqrt{-3}}{2}$ and

$$f(x) = \frac{1 - ix}{1 + ix} - (a - ib).$$

a. Find the square root of $(2 - 7i)$ 2

b. Show that $p^n + q^n = 2$ or -1 , where it is divisible by 3 or n is any other natural number respectively. 4

c. If $a, b, \in \mathbb{R}$ and $a^2 + b^2 - 1 = 0$, then show that the equation $f(x) = 0$ has a real root. 4

3. ► Suppose $F(x) = ax^2 + bx + c$, $G(x) = cx^2 + bx + a$ and $H(x) = 2x^2 - 2(a + b)x + a^2 + b^2$.

a. For which value of p , $(3p + 1)x^2 + (11 + p)x + 9 = 0$ has complex roots? 2

b. If one of the roots of $F(x) = 0$ is double of one of the roots of $G(x) = 0$, then show that, $2a = c$ or $(2a + c)^2 = 2b^2$. 4

c. Show that roots of $H(x) = 0$ can't be real if $a \neq b$. 4

4. ► Equation-1 : $4 \sin\theta \cos\theta - 1 + 2\sin\theta = 0$

Equation-2 : $\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} = 2 \tan^{-1} x$.

a. Find the value of $\cot \cos^{-1} \sin \tan^{-1} \left(\frac{3}{4} \right)$. 2

b. Solve the equation - 1 for $0 < \theta < \pi$. 4

c. From equation-2, show that $x = \frac{a-b}{a+ab}$. 4

Group B - Geometry, Mechanics and Statistics

5. ★

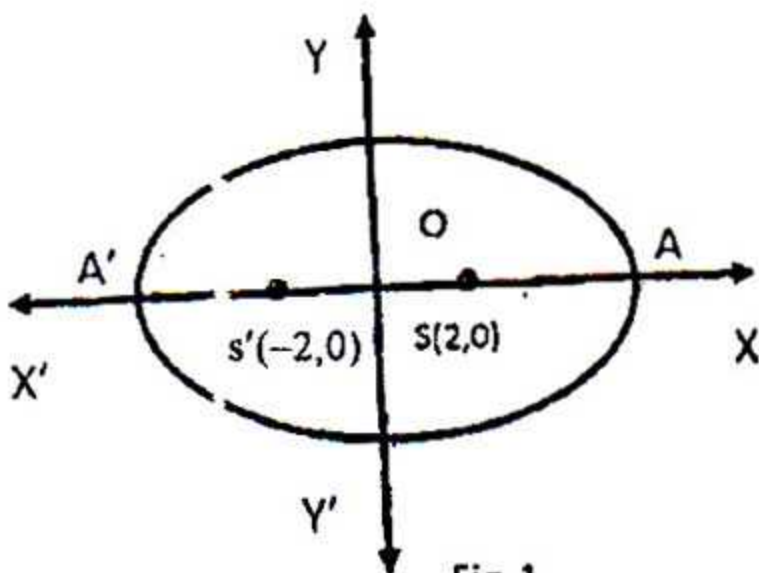


Fig-1

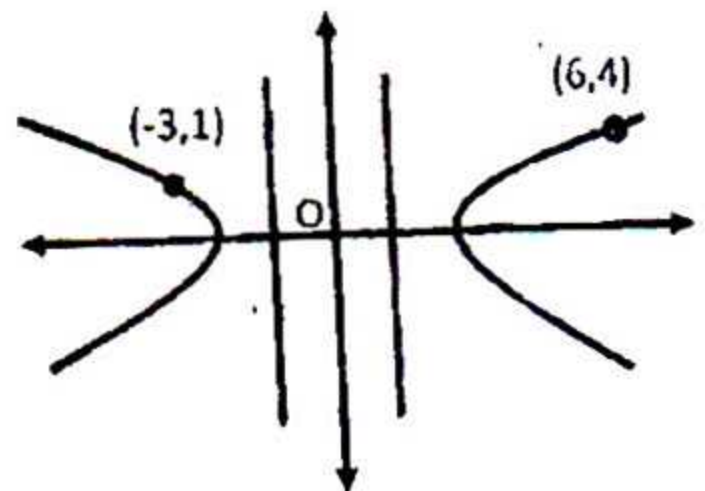


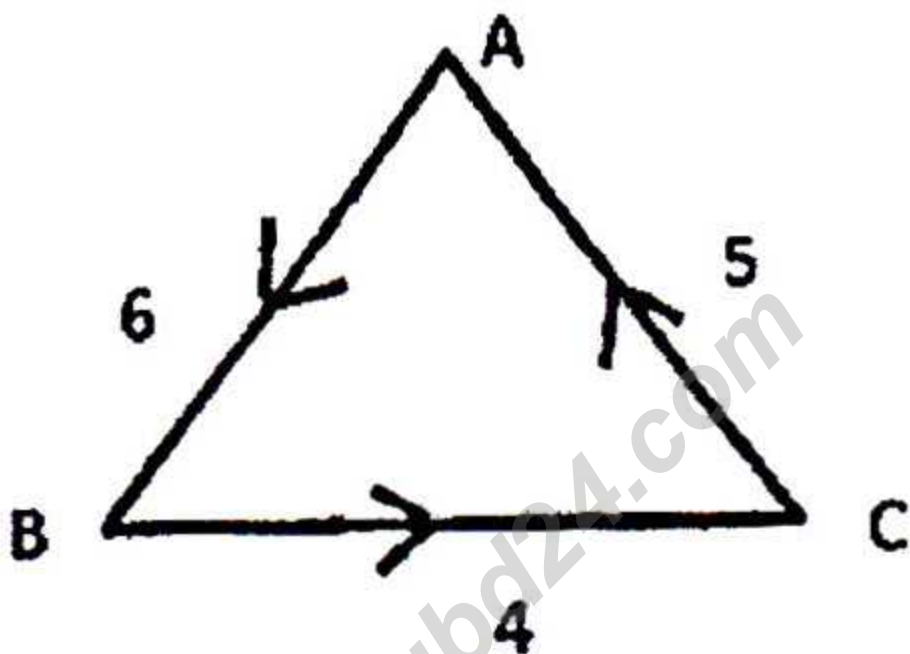
Fig-2

- a. Determine the equation of axis of the conic $x^2 = 4(1 - y)$. 2
- b. If in figure-1, $AA' = 8$ units, determine the equation of the conic. 4
- c. Determine the equation of the conic shown in the figure-2. 4
6. ★ Two forces P and Q ($P > Q$) are acting at point A and B of a rod as shown in the figure.



- a. What is the nature of the forces P and Q ? 2
- b. Find the resultant and the position of the resultant of P and Q . 4
- c. If $P = 8$ dyne, $Q = 3$ dyne, $AB = 12$ cm and the rod is kept balanced with only one another force, then find the minimum length of the rod. 4
7. ★ Scenario-1 : A man swims directly across a flowing river in time t_1 and he swims a distance equal to the breadth of the river down the stream in time t_2 . Here u is the speed of the man and v that of the stream ($u > v$)

Scenario-2 : Three velocities of magnitudes 4, 5 and 6 acting at a point along the directions shown in the equilateral triangle ABC.



- a. Show the information in the scenario-1 using a figure. 2
 - b. From scenario-1, prove that, $t_1 : t_2 = \sqrt{u + v} : \sqrt{u - v}$. 4
 - c. Find the magnitude and direction of the resultant of the velocities shown in scenario-2. 4
8. ► There are 6 red and 10 white balls in a box. A boy picks up two balls randomly one after another.
- a. Find the probability of getting two red balls. 2
 - b. Find the probability of getting two different color balls. 4
 - c. Find the probability of getting two same color balls. 4

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. Which one is the solution of the inequality $\left|5 - \frac{2}{3x}\right| < 1$?
- (a) $\frac{1}{9} < x < \frac{1}{6}$ (b) $-\frac{1}{9} < x < \frac{1}{6}$
 (c) $\frac{1}{9} < x < -\frac{1}{6}$ (d) $-\frac{1}{9} < x < -\frac{1}{6}$
2. \star If $-7 < x < -1$, which one is correct?
 (a) $|x + 4| < 1$ (b) $|x + 4| < 2$
 (c) $|x - 4| < 3$ (d) $|x + 4| < 3$
3. If $f(x) = |x - 3|$ and $f(x) = 7$, what is the value of x ?
 (a) 10, 4 (b) 10, -4
 (c) -10, 4 (d) -10, -4
4. A number of two digits is formed so that the sum of two digits is at least 12. Again if 2 is added to any of the digits, then the sum is of single digit. Which is the smallest number?
 (a) 57 (b) 67
 (c) 87 (d) 97
5. Roots of $x^2 - 2x - 1 = 0$ will be —
 i. Real
 ii. Rational
 iii. Unequal
 Which one is correct?
 (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii
6. What is the argument of $(-1 + i)$?
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
 (c) $\frac{3\pi}{4}$ (d) π
7. $\frac{i^{-1} - i}{2i^{-1} + i} = ?$
 (a) 1 (b) 2
 (c) 3 (d) 4
8. If $a = \frac{1+i}{\sqrt{2}}$, $a^6 = ?$
 (a) i (b) $-i$
 (c) 1 (d) -1
9. What is the minimum value of $x^2 - 2x + 5$?
 (a) 2 (b) 3
 (c) 4 (d) 5
10. If p, q be the roots of the equation $x^2 - 7x + 12 = 0$, which one is the equation with roots $p + q$ and pq ?
 (a) $x^2 - 19x + 84 = 0$
 (b) $x^2 + 14x - 144 = 0$
 (c) $x^2 - 14x + 144 = 0$
 (d) $x^2 + 19x - 84 = 0$
11. If $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ have one common root, then what is the value of a ?
 (a) 0, 24 (b) 1, -1
 (c) -1, 1 (d) 0, -24
12. If the coefficients of x^3 and x^4 are equal in the expansion of $(1 + ax)^8$, then $a = ?$
 (a) $\frac{1}{5}$ (b) $\frac{2}{5}$
 (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
13. \star What is the eccentricity of conic $3x^2 + 5y^2 = 15$?
 (a) $\frac{2}{5}$ (b) $\frac{\sqrt{2}}{5}$
 (c) $\frac{2}{\sqrt{5}}$ (d) $\sqrt{\frac{2}{5}}$

14. If $\cos\theta + \sqrt{3}\sin\theta = 2$,
and $(0^\circ < \theta < 360^\circ)$;
then what is the value of θ ?
 (a) 30° (b) 45°
 (c) 60° (d) 90°
15. Which one is the focus of the parabola
 $y^2 = -(x - 1)$?
 (a) $(0, \frac{3}{4})$ (b) $(\frac{3}{4}, 0)$
 (c) $(-1, 0)$ (d) $(0, -1)$
16. Which one is the vertex of the conic
 $y^2 - 4x - 8y = 0$?
 (a) $(4, 4)$ (b) $(4, -4)$
 (c) $(-4, -4)$ (d) $(-4, 4)$
17. If $\cos^{-1}\frac{2}{\sqrt{5}} = \sin^{-1}x$,
then what is the value of x ?
 (a) $\frac{-1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{5}}$
 (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-2}{\sqrt{5}}$
18. A person goes from A to B at a speed
of 4 km/h and returns from B to A at
a speed of 5 km/h.
What is his average velocity?
 (a) 4.67 km/h (b) 4.5 km/h
 (c) 4.44 km/h (d) 4.25 km/h
19. A person is carrying a weight W at one
edge of a stick which is kept on his
shoulder and the other edge of the
stick is held by his hand, if the
pressure on the shoulder is thrice the
weight of the object, then find the
distance of the hand from his shoulder.
 (a) 1 feet (b) 2 feet
 (c) 3 feet (d) 4 feet
20. The probabilities of solving a problem
by A and B $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What
is the probability of their solving it
together?
 (a) $\frac{1}{12}$ (b) $\frac{7}{12}$
 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$
21. Which one is the necessary and
sufficient condition for not being two
events A and B mutually exclusive?
 (a) $A \cap B = \phi$ (b) $A \cap B \neq \phi$
 (c) $A \cup B = \phi$ (d) $A \cup B \neq \phi$
22. What is the coefficient of x^2 in the $\frac{1+x}{1-x}$
?
 (a) 0 (b) 1
 (c) 2 (d) 3
23. What is the equation of latus rectum of
the conic $3y^2 - x^2 = 9$?
 (a) $y = 2\sqrt{3}$ (b) $y = -2\sqrt{3}$
 (c) $y = \pm 2\sqrt{3}$ (d) $y = 2\sqrt{3}$
24. Which is a real cubic root of (-1) ?
 (a) 1 (b) -1
 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$
25. Which quadratic equation has one
root $\sqrt{-5} - 1$?
 (a) $x^2 - 2x + 6 = 0$ (b) $x^2 - 2x + 5 = 0$
 (c) $x^2 - 2x - 5 = 0$ (d) $x^2 + 2x + 6 = 0$

Ans.	1	(a)	2	(d)	3	(b)	4	(a)	5	(b)	6	(c)	7	(b)	8	(b)	9	(c)	10	(a)	11	(a)	12	(d)	13	(d)	14	(c)	15	(b)
	16	(d)	17	(b)	18	(c)	19	(b)	20	(d)	21	(b)	22	(c)	23	(a)	24	(b)	25	(d)										