Model Question of HSC Examination 2020

Higher Mathematics 2nd Paper (Creative) Subject Code : | 2 | 6 | 6 |

Time — 2 hours 35 minutes | Full marks — 50

[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]

Group A – Algebra & Trigonometry

1. ✪ $z = 3x + 4y$, constraints : $x \le 2y + 2$, $x \ge 6 - 2y$, $y \le x$, $x \le 6$.

a. If $y = 1$ and $|z| < 1$, find the limits of x. 2

b. Find the minimum value of the objective function z under the given constraints. 4

c. If $x = 1$, $y = \sqrt{-1}$ and $\frac{z}{\bar{z}} = A + iB$, find argument of $A - iB$ 4

2. ▶ z_1 and z_2 are the two square roots of $z = 4 + 3i$.

a. If ω is a cube root of unity, show that,
$(1 + \omega - \omega^2)(\omega + \omega^2 - 1)(\omega^2 + 1 - \omega) = -8$ 2

b. Find a quadratic equation whose one root is $\frac{1}{z}$. 4

c. Find the distance between the points indicated by z_1 and z_2 in the Argond's diagram. 4

3. ▶ $f(x) = x^2 + 2x + 2$

a. If $f(x) = 0$, show that $x = -1 \pm \sqrt{-1}$ 2

b. Rewrite the inequality $f(x) < 10$ using absolute value sign. 4

c. For $n \in \mathbb{N}$ prove that coefficient of x^3 in the expansion of $\{f(x)\}^n$ is $\frac{2^{n-1}}{3} n(n^2-1)$ 4

4. ▶ $f(x) = \text{cosec } x$, $g(x) = \tan x$

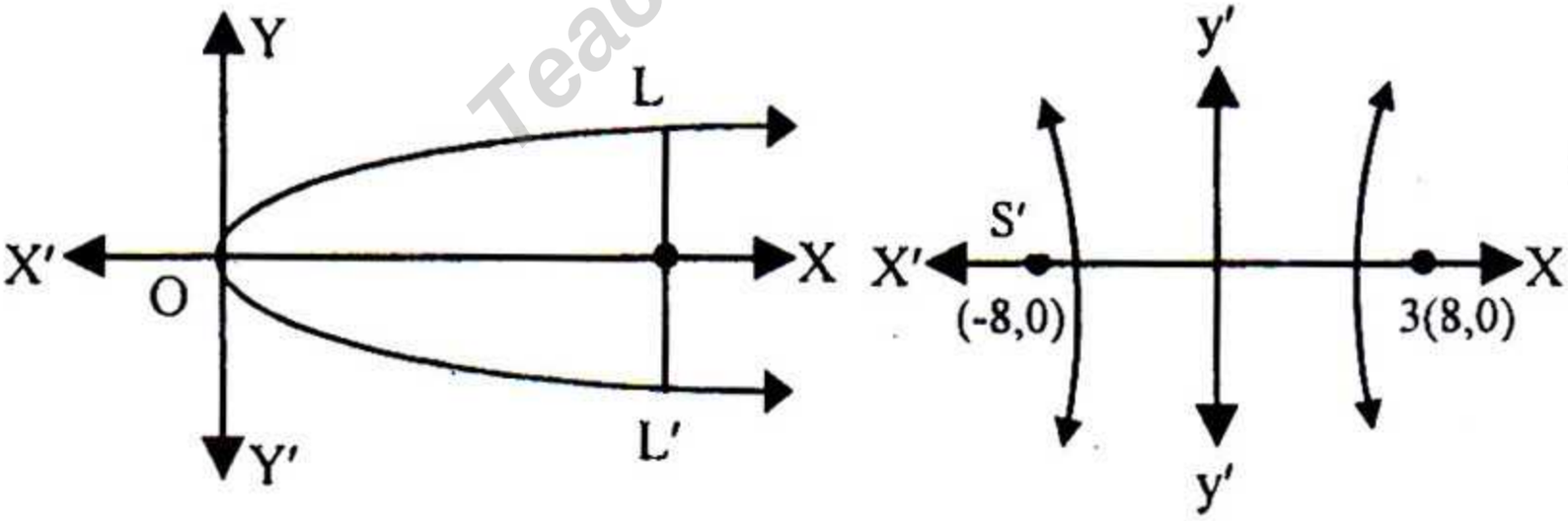
a. If $x = \frac{1}{2}\cos^{-1}\frac{3}{5}$, prove that $\tan x = \frac{1}{2}$. 2

b. Find the general solution of $2\tan^{-1}\{f(x)\} = \cot^{-1}\left(\frac{\cos x}{2}\right)$ 4

c. $\tan^{-1}\{f(\cos^{-1}x)\} - \tan^{-1}\{g(\sin^{-1}x)\} = \tan^{-1}\frac{(1-x)\sqrt{1-x^2}}{1+x-x^2}$. 4

Group B - Geometry, Mechanics and Statistics

5. ✪

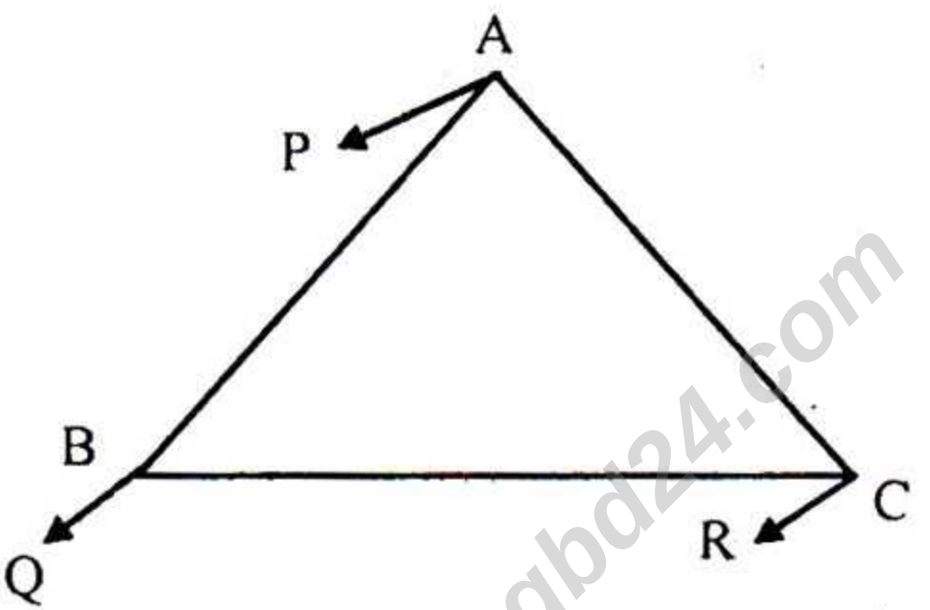


a. Find the equation of a hyperbola which passes through the point (5, 9) and whose asymptote are $y = \pm x$. 2

b. In the stem, LL′ is the latus rectum of a parabola and area of ΔOLL′ is 16 square units; find its equation. 4

c. Find out the equation of the hyperbola if S′ and S are its two foci started in the stem. and the distance of a directrix from its centre is 4 units. 4

6. ✪



P, Q, R are three like parallel forces.

a. Two forces P and Q acting at a point have a resultant R. If Q be double, then the new resultant is perpendicular to the line of P. Prove that Q = R. 2

b. If the resultant of the forces started in the figure of the stem passes through the orthocenter of the triangle, prove that, P : Q : R = tan A : tan B : tan C. 4

c. If AB = BC = CA and forces S – T, S, S + T act at a point in directions parallel to BC, CA, AB respectively, find the magnitude of their resultant. 4

7. ▶ Scenario-1 : An aeroplane moving with uniform velocity of 50 km/h touches a straight runway and comes at rest after describing a distance 300 metres.

Scenario-2 : A particle is projected with a velocity $98ms^{-1}$ at an angle 60° with the horizon.

a. A bomb falling on the ground burst out and its particles move with a velocity u in all directions on a horizontal plane. Show that the particles fall within a circle. Find the area of this circle. 2

b. If the retardation is uniform in the light f scenario-1, find the time to come at rest. 4

c. Find the magnitude and direction of the velocity of the particle stated in scenario-2 at a height of 320m. 4

8. ▶ A frequency distribution table is given below :

Class interval	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
Frequency	7	10	15	13	9	6

a. Find the range from the given frequency distribution table. 2

b. Find the variance from the given frequency distribution table. 4

c. Find the quartile deviation from the given frequency distribution table. 4

Higher Mathematics 2nd Paper (MCQ)

Subject Code : 2 6 6

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. If $S = \{x \in \mathbb{R} : -2 \le 3 - x \le 8\}$,

i. $|x| \le 5$

ii. $S = \{x \in \mathbb{R} : x \in [-5, 5]\}$

iii. $\inf S = 5$

Which one is correct?

ⓐ i ⓑ ii

ⓒ i & ii ⓓ i, ii & iii

2. Which one is correct?

ⓐ $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{C} \subset \mathbb{R}$

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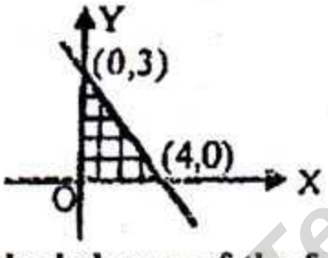
ⓓ $\mathbb{N} \subset \mathbb{Q} \subset \mathbb{Z} \subset \mathbb{C} \subset \mathbb{R}$

3. Which one is correct for $a, b \in \mathbb{R}$?

ⓐ $|a - b| > |a| + |b|$ ⓑ $|a + b| < |a| + |b|$

ⓒ $|a + b| = |a| + |b|$ ⓓ $|a - b| = |a| + |b|$

Ansawer the questions no. 4 and 5 according to the following stem :



4. ★ If the shaded area of the fesible region of a linear programming, what are the conditions?

ⓐ $4x + 3y \ge 24;\ x, y > 0$

ⓑ $3x + 4y \le 24;\ x, y > 0$

ⓒ $4x + 3y \ge 24;\ x, y \ge 0$

ⓓ $3x + 4y \le 24;\ x, y \ge 0$

5. ★ If objective function of the linear program is $z = 3x + 2y$, what will be the maximum value of z?

ⓐ 6 ⓑ 8

ⓒ 12 ⓓ 24

6. What is the argument of $-\sqrt{3} - i$?

ⓐ $-\frac{2\pi}{3}$ ⓑ $\frac{\pi}{6}$

ⓒ $\frac{7\pi}{6}$ ⓓ $-\frac{5\pi}{6}$

7. What is the value of $(1 + \omega)^{2019}$?

ⓐ 1 ⓑ -1

ⓒ $-\omega$ ⓓ ω

8. $i + i^{-1} + i^{-2} + i^{-3}$ = what?

ⓐ -1 ⓑ $-i$

ⓒ 0 ⓓ i

9. If roots of the equation $3x^3 - 9x^2 - 6x + 5 = 0$ are α, β and γ, what is the value of $\sum \alpha\beta$?

ⓐ -3 ⓑ 3

ⓒ -2 ⓓ 2

10. ★ If α, β are the roots of $7x^2 - 5x - 3 = 10$, then

i. $\alpha + \beta = \frac{5}{7}$

ii. $\alpha\beta = \frac{3}{7}$

iii. The equation having roots $\alpha + 1$ and $\beta + 1$ is $7x^2 - 19x - 9 = 0$

Which one is correct?

ⓐ i ⓑ ii

ⓒ i & ii ⓓ i & iii

Answer the question 11 and 12 according to the stem: $\left(x^2 - \frac{1}{x}\right)^{16}$ is an expansion.

11. What is the value of middle term?

ⓐ ${}^{16}C_8$ ⓑ ${}^{16}C_8x^8$

ⓒ $-{}^{16}C_9x^5$ ⓓ $-{}^{16}C_9x^{-2}$

12. What is the 10^{th} term to the left from the end?

ⓐ $-{}^{16}C_9x^5$ ⓑ ${}^{16}C_8x^8$

ⓒ $-{}^{16}C_7x^2$ ⓓ $-{}^{16}C_7x^{11}$

13. In the expansion of $\frac{1}{\sqrt{1 - 7x}}$ coefficient of x^2 is —

ⓐ $-\frac{147}{4}$ ⓑ $-\frac{147}{8}$

ⓒ $\frac{147}{8}$ ⓓ $\frac{147}{4}$

14. $\sin(2\sin^{-1}\frac{1}{2})$ = what?

ⓐ $\frac{1}{4}$ ⓑ $\frac{\sqrt{3}}{4}$

ⓒ $\frac{\sqrt{3}}{2}$ ⓓ $\frac{\sqrt{5}}{2}$

15. General solution of $\cos\theta + \sqrt{3}\sin\theta = 2$ is –

ⓐ $2n\pi - \frac{\pi}{3}$ ⓑ $2n\pi - \frac{\pi}{6}$

ⓒ $2n\pi + \frac{\pi}{3}$ ⓓ $2n\pi + \frac{\pi}{6}$

16. Value of $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$ is —

ⓐ 0 ⓑ $\frac{\pi}{2}$

ⓒ π ⓓ $\frac{3\pi}{2}$

17. Value of arc tan {sin (arc cos $\frac{\sqrt{2}}{\sqrt{3}}$)} is—

ⓐ $\tan^{-1}\left(\frac{1}{3}\right)$ ⓑ $\tan^{-1}\left(\sqrt{\frac{5}{3}}\right)$

ⓒ $\frac{\pi}{3}$ ⓓ $\frac{\pi}{6}$

18. Two forces equal to 2p and P act on a particle. If the first be doubled and the second increased by 8 units, the direction of the resultant is unaltered. The value of P is —

ⓐ 16 ⓑ 8

ⓒ 4 ⓓ 2

19. Two equal forces are acting on a particle at an angle 90°. If the two forces with the force $(\sqrt{3}+\sqrt{2})$ Newton are in equilibrium, what is the value of each of them?

ⓐ $\frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{2}}$ ⓑ $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}}$

ⓒ $\frac{\sqrt{5+2\sqrt{6}}}{2}$ ⓓ $\frac{5+2\sqrt{6}}{\sqrt{2}}$

20. A weight of 9 kg is suspended at an end of a uniform rod of length 12m. This rod rests horizontally on a prop at a distance 5.25 m from this end. Which one of the following is the weight of the rod?

ⓐ 1.29 kg ⓑ 6.1 kg

ⓒ 13.29 kg ⓓ 63 kg

21. What is the vertex of the conic $16x^2 - 9y^2 + 144 = 0$?

ⓐ $(0, \pm 4)$ ⓑ $(0, \pm 5)$

ⓒ $(\pm 4, 0)$ ⓓ $(\pm 5, 0)$

22. If ordinate of a point on the parabola $y^2 = 9x$ is 12, the focal distance of the point is —

ⓐ 9.50 ⓑ 18.25

ⓒ 10.50 ⓓ 20.25

23. In the ellipse $\frac{(x-3)^2}{3} + \frac{(y+1)^2}{4} = 1$

i. coordinate of a vertex is (3, – 1)

ii. length of minor axis is 6

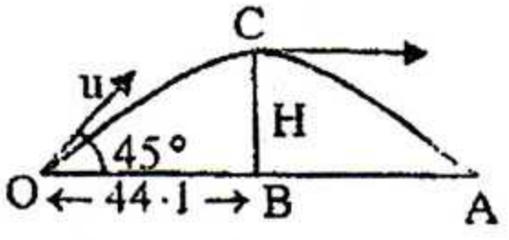
iii. one equation of the lateral recta is y+2=0

Which one is correct?

ⓐ i & ii ⓑ ii & iii

ⓒ i & iii ⓓ i, ii & iii

24. ★



According to the figure what is the value of u; when $g = 9.8\ ms^{-2}$?

ⓐ 20 ms^{-1} ⓑ 22 ms^{-1}

ⓒ 25.4 ms^{-1} ⓓ 29.4 ms^{-1}

25. A car starting with initial velocity 15 ms^{-1} and uniform acceleration 4 ms^{-2} crosses a pillar at a distance 15m from the starting point. What is the velocity of the train when it crosses the pillar?

ⓐ 37.75 ms^{-1} ⓑ 30.75 ms^{-1}

ⓒ 29.75 ms^{-1} ⓓ 28.75 ms^{-1}

Ans.																														
	1	ⓒ	2	ⓒ	3	ⓑ	4	ⓓ	5	ⓒ	6	ⓓ	7	ⓑ	8	ⓒ	9	ⓒ	10	ⓐ	11	ⓑ	12	ⓓ	13	ⓒ	14	ⓒ	15	ⓒ
	16	ⓒ	17	ⓓ	18	ⓑ	19	ⓑ	20	ⓓ	21	ⓐ	22	ⓑ	23	ⓒ	24	ⓓ	25	ⓐ										