

Model Question of HSC Examination 2020

Higher Mathematics 2nd Paper (Creative) Subject Code :

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Full marks — 50

Time — 2 hours 35 minutes

[N.B. — Right marking indicate the full marks, taking at least two from each group answer the five questions]

Group A – Algebra & Trigonometry

1. ► $f(x) = 27x^2 + 6x - (a + 2)$ and $g(x) = x^2 + px + q$.

a. For what value of 'k' the roots of the equation $(k - 1)x^2 - (k + 2)x + 4 = 0$ will be real and equal? 2

b. If the difference of the roots of $g(x) = 0$ is unity, then prove that $p^2 + 4q^2 = (1 + 2q)^2$. 4

c. If one root of $f(x) = 0$ is square of another, then find the value of 'a'. 4

2. ★ $Z = -1 + i\sqrt{3}$ and $Z_1 = a + ib$

a. For all $p, q \in \mathbb{R}$, prove that $|p + q| \leq |p| + |q|$ 2

b. Prove that, $(Z)^4 + (\bar{Z})^4 = -16$ 4

c. If $\sqrt[3]{Z_1} = x + iy$ then show that $4(x^2 - y^2) = \frac{a}{x} + \frac{b}{y}$ 4

3. ► $P = (a + bx)^n$.

a. If $y = x + x^2 + x^3 + \dots$, then prove that $x = y - y^2 + y^3 - y^4 + \dots$ 2

b. If $a = b = 1$, then find the term independent of x in the expansion of $P \left(1 + \frac{1}{x}\right)^m$ where $m, n \in \mathbb{N}$. 4

c. If $a = 1$, $b = -4$ and $n = -\frac{1}{2}$, then find the coefficient of x^r in the expansion of P . 4

4. ► $f(x) = \sin x$ and $g(x) = \cos x$

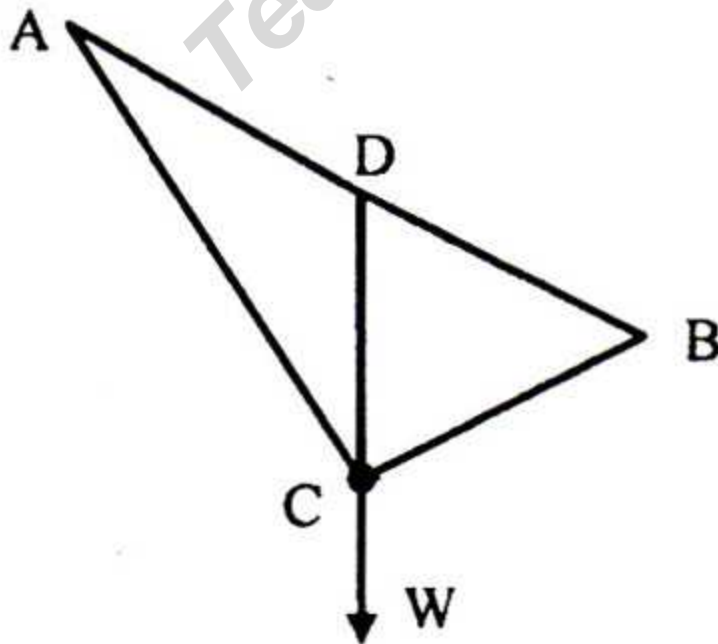
a. Prove that, $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) = 15$. 2

b. If $f(\pi \cos \theta) = f\left(\frac{\pi}{2} - \pi \sin \theta\right)$, then prove that $\theta = \pm \frac{1}{2} \sin^{-1} \frac{3}{4}$. 4

c. Solve : $\frac{1}{g(4x)} - \frac{1}{g(2x)} = 2$, where $0^\circ < x < 2\pi$. 4

Group B – Mechanics, Conics and Dispersion of Probability

5. ★



The weight W is kotted at C & $AB = c$, $AC = b$, $BC = a$ where D is the middle point of AB .

a. If the greatest and least resultant of two forces are 9N and 4N , then what are the forces? 2

b. If $a = 16\text{m}$, $b = 12\text{m}$, $c = 20\text{m}$, $W = 30\text{kg-wt}$, then find the tension of AC and BC . 4

c. If AB is horizontal and Δ be the area of the triangle ABC , then show that the tension of the rope CA is $\frac{Wb}{4c\Delta} (c^2 + a^2 - b^2)$. 4

6. ★ A cricket ball is projected with the velocity u m/s and another ball is projected with the same velocity.

a. A bullet fired into a target with velocity 8m/s losses half of its velocity after penetrating 2m . What is its retardation? 2

b. If the 2nd ball is projected after n seconds of the 1st and both the ball are projected vertically upwards, then show that they will meet at a distance $\frac{4u^2 - g^2n^2}{8g}$ from the point of projection. 4

c. If the 1st ball is projected with an angle α and the 2nd ball is projected with an angle β and both the ball cross same horizontal distance, then prove that $\alpha = \frac{\pi}{2} - \beta$. 4

7. ► $x + y - 2 = 0$ (i), $y = px^2 + qx + r$ (ii), and $S(3, 4)$.

a. For what of k , the line $y = k - 2x$ will be a tangent to the curve $xy = 1$. 2

b. If (ii) passes through $(0, 5)$ and its vertex is at $(-2, 3)$, then find the value of p, q, r . 4

c. Find the equation of the ellipse taking S as the focus, (i) as directrix and whose eccentricity is $e = \frac{1}{3}$. 4

8. ► Among three bags, first bag contains 5 white and 4 red balls; second bag contains 3 white and 7 red balls; third bag contains 6 white, 7 red and 5 black balls.

a. Determine the variance of first n natural number. 2

b. If three balls are drawn at random from the third bag, then what is the probability that two of them are same color. 4

c. A bag is selected in equiprobable way and two balls are drawn from the bag. Find the probability that one ball is red and another ball is white. 4

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. $(1 + \omega^{3n+1})(1 + \omega^{3n+2}) = \text{What?}$
 (a) 0 (b) 1
 (c) ω (d) ω^2
2. If $S = \left\{ \frac{3n+2}{2n+1}, n \in \mathbb{N} \right\}$, then the —
 i. set S is bounded
 ii. $\sup S = \frac{5}{3}$
 iii. set of all upper bounded is $x \geq \frac{5}{3}$
 Which one of the following is correct?
 (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii
3. Which one is the maximum value of $Z = 2x + y$ under the constraints $x + y \leq 6$, $x < 4$, $y \leq 5$ and $x, y > 0$?
 (a) 7 (b) 8
 (c) 9 (d) 10
4. The principal argument of $z: = -i - \sqrt{3}$ is —
 (a) $-\frac{2\pi}{3}$ (b) $-\frac{5\pi}{6}$
 (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$
5. For which condition the equation $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have both the roots common?
 (a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 (c) $\frac{a_1}{b_1} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
6. If α, β be the roots of the equation $px^2 + qx + r = 0$, then the —
 i. sum of the root is $-\frac{q}{p}$
 ii. Product of the root is $\frac{r}{p}$
 iii. difference of the roots is $\frac{\sqrt{q^2 - 4pr}}{p}$
 Which one of the following is correct?
 (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii
7. The infimum of the set $S = \{x \in \mathbb{R}; 5x^2 - 16x + 3 < 0\}$ is —
 (a) 3 (b) 5
 (c) $\frac{1}{3}$ (d) $\frac{1}{5}$
8. $C_1 + C_2 + C_3 + \dots + C_n = \text{what?}$
 (a) 2^n (b) 2^{n-1}
 (c) $2^n - 1$ (d) $2^n + 1$
9. If $x = \frac{1}{3}$, then which term is numerically greatest in the expansion of $(2+5x)^{10}$?
 (a) 5th term (b) 6th term
 (c) 5th & 6th term (d) 6th & 7th term
10. In the expansion of $(a+x)^{30}$, the —
 i. number of term is 31.
 ii. middle term is ${}^{30}C_{15} (ax)^{15}$
 iii. 12th term from the end is ${}^{30}C_{11} a^{19}x^{11}$
 Which one of the following is correct?
 (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii
11. For which limit the binomial expansion $(3-4x)^{\frac{1}{2}}$ will be convergent?
 (a) $|x| < \frac{3}{4}$ (b) $|x| < \frac{4}{3}$
 (c) $|x| < \frac{1}{2}$ (d) $|x| < 1$
12. The equation $11x^2 + 11y^2 - 24xy - 50x - 225 = 0$ represents—
 (a) Circle (b) Ellipse
 (c) Parabola (d) Hyperbola.
- Answer to the question number (13 – 14) with the help of the following information:
 (0, 0) and (1, 1) are the coordinate of two foci and (-1, -1) is a point on the ellipse.
13. Which one is the equation of minor axis?
 (a) $x + y - 1 = 0$ (b) $x - y + 1 = 0$
 (c) $x - y = 0$ (d) $x + y = 0$

14. For the ellipse, the —
 i. distance between two foci is $\sqrt{2}$ unit
 ii. length of the major axis is $2\sqrt{3}$ unit.
 iii. eccentricity is $\frac{2}{3}$.

Which one of the following is correct?

- (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii

15. $2 \tan^{-1} x =$ what?

- (a) $\cot^{-1} \frac{1-x^2}{2x}$ (b) $\tan^{-1} \frac{2x}{1+x^2}$
 (c) $\cos^{-1} \frac{1+x^2}{1-x^2}$ (d) $\sin^{-1} \frac{2x}{1-x^2}$

16. $\sin^{-1} \frac{3}{4} + \sec^{-1} \frac{4}{3} =$ what?

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

17. If $\sin \theta = 1$, then $\theta =$ what?

- (a) $(2n+1)\frac{\pi}{2}$ (b) $(2n-1)\frac{\pi}{2}$
 (c) $(4+1)\frac{\pi}{2}$ (d) $(4n-1)\frac{\pi}{2}$

Answer to the question number (18 – 19) with the help of the following information:

A bullet is seen to pass horizontally just over a vertical wall of height 75 ft and 50 yards away from the point of projection.

18. Which one is the angle of projection?

- (a) 30° (b) 45°
 (c) 60° (d) 75°

19. For the bullet, the —

- i. greatest height is 75 ft.
 ii. range is 100 ft.
 iii. time of flight is 4.33 s.

Which one of the following is correct?

- (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii

20. There are three children in a family and one of them is at least a boy.

What is the probability that there are two boys in the family?

- (a) $\frac{2}{3}$ (b) $\frac{2}{7}$
 (c) $\frac{3}{7}$ (d) $\frac{6}{7}$

21. Out of 120 students, 75 play Cricket and 65 play Football. If one student is selected at random then what is the probability that he plays both the game?

- (a) $\frac{1}{12}$ (b) $\frac{1}{6}$
 (c) $\frac{5}{8}$ (d) $\frac{13}{24}$

22. For the data 5, 7, 0, -3, 11, 25, 17, 4, 20, 26, the —

- i. 1st quartile is 4
 ii. 3rd quartile is 20
 iii. coefficient of quartile deviation is 66.67%.

Which one of the following is correct?

- (a) i & ii (b) i & iii
 (c) ii & iii (d) i, ii & iii

23. If the resultant of two forces $\sqrt{3}p$ and $2p$ is perpendicular to $\sqrt{3}p$, then what is the angle between the forces?

- (a) 60° (b) 90°
 (c) 120° (d) 150°

24. Two like parallel forces 9N and 6N are acting at A and B where $AB = 15$ cm. If the position of the forces are interchanged then the resultant will displace —

- (a) 1 cm (b) 2 cm
 (c) 3 cm (d) 6 cm

25. A balloon is ascending vertically with a constant velocity 50 ft/s and a stone is let fall from it which reaches the ground in 10 sec. At which velocity the stone hits the ground?

- (a) 48 ft/s (b) 50 ft/s
 (c) 148 ft/s (d) 270 ft/s

Ans.	1	(b)	2	(d)	3	(a)	4	(d)	5	(b)	6	(a)	7	(d)	8	(c)	9	(d)	10	(a)	11	(a)	12	(d)	13	(a)	14	(a)	15	(a)
	16	(a)	17	(c)	18	(b)	19	(b)	20	(c)	21	(b)	22	(d)	23	(d)	24	(d)	25	(d)										