

# Dinajpur Board-2017

Higher Mathematics 1<sup>st</sup> Paper (Creative) Subject Code : 

2	6	5
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Time — 2 hours 35 minutes

Full marks — 50

*[N.B. The figures in the right margin indicate full marks.  
Answer five questions taking at least two from each group.]*

## Group A – Algebra and Geometry

1. ★  $M = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -3 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

a. If  $\begin{bmatrix} 2 & -x \\ y-1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3+y \\ 4 & 2 \end{bmatrix}$  then find  $(x, y)$ . 2

b. Find  $M^2 - 3M + MI$ , where  $I$  is an identity matrix. 4

c. If  $M^{-1}$  exists, find it. 4

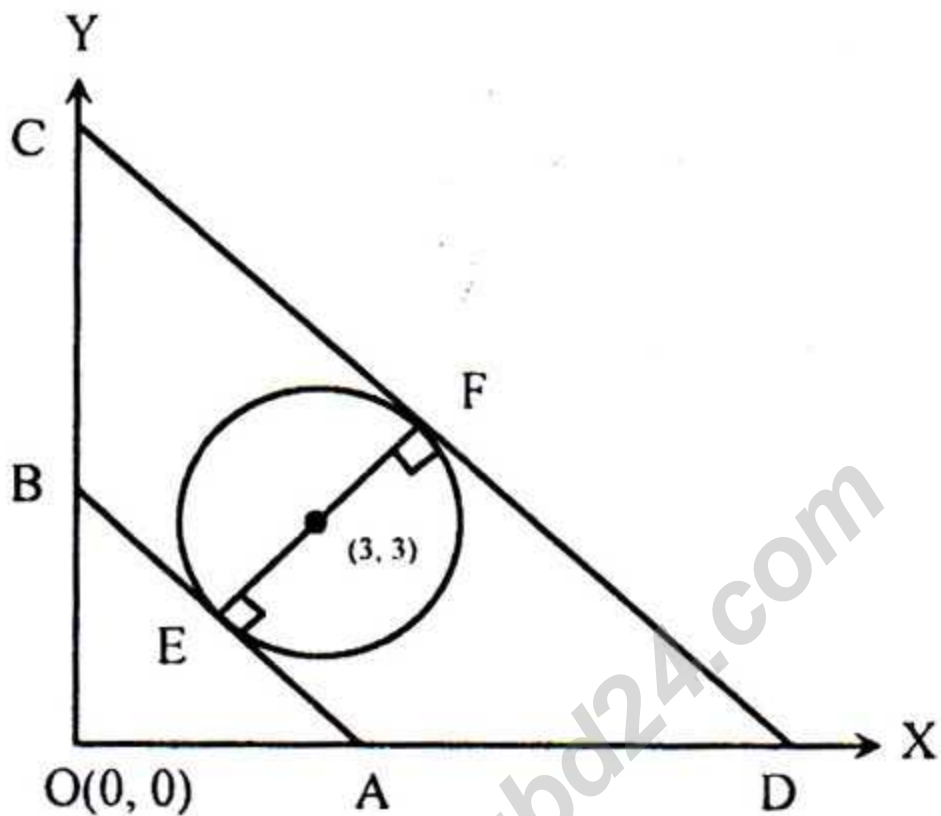
2. ►  $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ ,  $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ ,  $\vec{C} = \hat{i} + b\hat{j} + 3\hat{k}$ .

a. What do you mean by position vector? 2

b. If the component of vector  $\vec{B}$  along  $\vec{A}$  is perpendicular to  $\vec{C}$ , then find the value of  $b$ . 4

c. Find the angle between the vectors  $\vec{A} + \vec{B}$  and  $\vec{A} \times \vec{B}$ . 4

3. ►



Figure

- Find the centre and radius of the circle,  $3(x^2 + y^2) - 5x + y + 1 = 0$ . 2
  - If  $OA = 4$  and  $OB = 3$ , then determine the equation of the circle drawn in the figure. 4
  - If  $AB \parallel CD$ , then find the equation of the circle whose terminal points of diameter are  $F$  and  $D$ . 4
4. ► In a Onesday match NEWZEALAND cricket team consists of 7 batsman, 6 bowlers and 2 wicket keepers.
- If  ${}^n P_3 = 2 \times {}^n C_4$ , then find the value of  $n$ . 2

- b. How many ways the cricket team of 11 players can be made so as always including 5 bowlers and at least one wicket keeper. 4
- c. Find the number of arrangements of the letters of the word NEWZEALAND not keeping the vowels side by side. 4

### Group B – Trigonometry and Calculus

5. ►  $f(x) = \sin x$  and  $g(x) = \cos x$ .

a. If  $\cos\theta = \frac{3}{\sqrt{13}}$ , then find the value of  $\sqrt{\frac{2 - \cot^2\theta}{2 + \cot^2\theta}}$ . 2

b. If  $f(x) + f(y) = p$  and  $g(x) + g(y) = q$ , then prove that  $f\left(\frac{x-y}{2}\right) = \pm \frac{1}{2} \sqrt{4 - p^2 - q^2}$ . 4

c. Draw the graph of  $f(2x)$  in the interval  $-\frac{\pi}{2} \leq x \leq \pi$  and write one characteristics of it. 4

6. ►  $A, B \subset \mathbb{R}$ ,  $B = \mathbb{R} - \left\{\frac{1}{3}\right\}$ ,  $g : A \rightarrow B$ ,  $g(x) = \frac{x-5}{3x+1}$

and  $h(x) = x^2 + 1$ .



- a. Determine the differential co-efficient of  $\sin e^{\sqrt{1-x}}$ . 2
- b. Show that  $(hog)(1) - (goh)(2) = 2$ . 4
- c. Verify the existence of  $g^{-1}(x)$ , if  $g^{-1}(x)$  exist, then find it. 4

7. ★  $f(x) = \ln x$  and  $g(x) = e^x$ .

- a. Find the value of  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{ax}$ . 2
- b. Determine the maximum and minimum value of  $\frac{f(2x)}{x}$  if it exists. 4
- c. Evaluate  $\int_1^{e^2} \frac{f(x)}{x} dx + \int_1^2 g(x) dx$ . 4

8. ★  $g(z) = mz \sin^{-1} z$  is a function and  $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$  is a curve.

- a. Find the value of  $\int_1^2 \frac{1}{z} \cos(\ln z) dz$ . 2
- b. Integrate  $\int g(x) dx$ . 4
- c. If  $b > a$  then find the half area of the region bounded by the curve given in the stem. 4

Time — 25 minutes

Full marks — 25

[N.B. Choose the best answer among the options. Fill the circle in the answer sheet with ball point pen. Each question has value 1.]

1. Which one of the following is correct?

- a.  $\sin(-\theta) = \sin\theta$       b.  $\cos(-\theta) = \cos\theta$   
 c.  $\tan(-\theta) = \tan\theta$       d.  $\cot(-\theta) = \cot\theta$

2. The angle between the vector  $2\hat{i} + 3\hat{j} - 6\hat{k}$  and the y-axis is —

- a.  $\cos^{-1}\left(\frac{2}{7}\right)$       b.  $\cos^{-1}\left(-\frac{2}{7}\right)$   
 c.  $\cos^{-1}\left(\frac{3}{7}\right)$       d.  $\cos^{-1}\left(-\frac{6}{7}\right)$

3. The period of the function  $y = \cos x$  is —

- a.  $\frac{\pi}{2}$       b.  $\pi$       c.  $-\pi$       d.  $2\pi$

4. i.  ${}^n P_n = n!$     ii.  ${}^n C_r = {}^n C_{n-r}$     iii.  ${}^n P_r = {}^n C_r \times r!$ 

Which one of the following is correct?

- a. i and ii      b. i and iii      c. ii and iii      d. i, ii and iii

5.  $\star$  The value of  $\lim_{x \rightarrow \infty} \frac{4+3x-x^2}{7+2x+3x^2}$  is —

- a.  $\frac{4}{7}$       b.  $\frac{3}{2}$       c.  $-\frac{1}{3}$       d.  $\frac{1}{3}$

Answer to the questions number 6 and 7 with the help of stem:

$$4x - 2y = 6$$

6. What is the intersecting point of x axis according to the stem?

- a.  $\left(\frac{3}{2}, 0\right)$       b.  $(0, -3)$       c.  $(-3, 0)$       d.  $\left(0, \frac{3}{2}\right)$

7. What is the slope to the stem?

- a. 2      b.  $-\frac{1}{2}$       c. -2      d.  $\frac{1}{2}$

8. i. There exists an inverse matrix of every non singular matrix  
 ii. If A and B are non singular matrices, then  $(AB)^{-1} = B^{-1}A^{-1}$   
 iii. The value of a determinant is changed if the corresponding rows and columns are interchanged

Which one of the following is correct?

- a. i and ii      b. i and iii      c. ii and iii      d. i, ii and iii

9. The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2 + 3$ , then which is the value of  $f^{-1}(11)$ ?

- a. 2      b.  $2\sqrt{2}$       c. 3      d.  $2\sqrt{3}$

10. The number of rearrangements made by the letters of the word 'DIFFERENT' so that the vowels do not change their positions is —

- a. 720      b. 719      c. 360      d. 359

11. For which value of m the vectors  $\vec{P} = 4\hat{i} + m\hat{j}$  and  $\vec{Q} = 6\hat{i} - 4\hat{j} + 3\hat{k}$  are perpendicular to each other?

- a. 4      b. 6      c. 8      d. -6

12.  $\star$  If A and B are two vectors, then —

- i.  $\vec{A} + \vec{B} = \vec{B} + \vec{A}$       ii.  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$   
 iii.  $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

Which one of the following is correct?

- a. i and ii      b. i and iii      c. ii and iii      d. i, ii and iii

13. i. The centre of the circle  $x^2 + y^2 = a^2$  is at the origin  
 ii.  $x^2 + 2y^2 = 4$  is an equation of a circle  
 iii. The circle  $(x+3)^2 + (y-2)^2 = 9$  touches the y-axis

Which one of the following is correct?

- a. i and ii      b. i and iii      c. ii and iii      d. i, ii and iii

14. The number of diagonals of a polygon of sides 16 is —

- a. 240      b. 224      c. 120      d. 104

15. The slope of the normal drawn at a point (2, 3) of the curve  $y = x^2 - x + 1$  is —

- a. 3      b. -3      c.  $\frac{1}{3}$       d.  $-\frac{1}{3}$

16. Inverse matrix of  $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$  is —

- a.  $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$       b.  $\frac{1}{2} \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$   
 c.  $\frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$       d.  $-\frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

Answer to the questions number 17 and 18 with the help of stem:

$$x^2 + y^2 + 4x - 6y + 1 = 0$$

17. What is the intersecting part of y axis according to the stem?

- a. 6      b.  $2\sqrt{2}$       c.  $4\sqrt{2}$       d. 0

18. Which one of the following is the radius of the stem?

- a. 2      b. 3      c.  $2\sqrt{3}$       d.  $\sqrt{14}$

19. In  $\triangle ABC$ ,  $\angle A = 90^\circ$ ,  $\angle B = 60^\circ$  and  $c = 3$  cm, then the length of b is —

- a. 6 cm.      b. 4 cm.      c.  $3\sqrt{3}$  cm.      d.  $2\sqrt{3}$  cm.

20. The polar co-ordinates of the point  $(-1, \sqrt{3})$  is —

- a.  $\left(2, \frac{\pi}{6}\right)$       b.  $\left(2, \frac{\pi}{4}\right)$       c.  $\left(2, \frac{\pi}{3}\right)$       d.  $\left(2, \frac{2\pi}{3}\right)$

21.  $\star$  If  $\frac{\pi}{2} < \theta < \pi$  and  $\sin\theta = \frac{3}{5}$ , then the value of  $\cos\theta$  is —

- a.  $\frac{3}{4}$       b.  $\frac{4}{5}$       c.  $-\frac{3}{4}$       d.  $-\frac{4}{5}$

22. The value of  $\frac{d^n}{dx^n}(x^n)$  is —

- a.  $n!$       b. x      c. 1      d. 0

23. The area bounded by the ellipse  $4x^2 + 25y^2 = 100$  is —

- a.  $4\pi$  sq. units      b.  $25\pi$  sq. units  
 c.  $10\pi$  sq. units      d.  $100\pi$  sq. units

24.  $\star$  i.  $\frac{d}{dx}(\log_a x) = \frac{1}{x} \log_a e$ 

ii.  $\int \sec^2 \frac{1}{2} x \, dx = \frac{1}{2} \tan \frac{1}{2} x + C$

iii.  $\int_a^b \frac{dx}{x} = \ln \left(\frac{b}{a}\right)$

Which one of the following is correct?

- a. i and ii      b. i and iii      c. ii and iii      d. i, ii and iii

25.  $\star$  The value of  $\begin{vmatrix} p & 2 & q+r \\ q & 2 & r+p \\ r & 2 & p+q \end{vmatrix}$  is —

- a. 0      b. 1  
 c. pqr      d.  $p+q+r$

Ans.	1	b	2	c	3	d	4	d	5	c	6	a	7	a	8	a	9	a	10	d	11	b	12	a	13	b
	14	d	15	d	16	b	17	c	18	c	19	c	20	d	21	d	22	a	23	c	24	b	25	a		