

Chattogram Board 2017

Higher Mathematics

Subject Code :

1	2	6
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Time — 2 hours 30 minutes

Full Marks — 50

[N.B. — The figures in the right margin indicate full marks. Answer five questions taking at least one from each Group.]

Group A — Algebra

1. ► $S = \{(x, y) : x^2 + y^2 + 6x + 8y + 9 = 0\}$ be a relation and $A = \{x : x \in \mathbb{N}, x \text{ is prime number and } x < 7\}$, $B = \{x : x \text{ is positive integer and } \sqrt{x} < 2\}$ are two sets.

- a. Express the set B in tabular method. 2
- b. Show that, $P(A) \cap P(B) = P(A \cap B)$ 4
- c. Draw the graph of the relation "S" and ascertain from the graph whether 'S' is a function or not. 4

2. ► $\frac{1}{3x+2} + \frac{1}{(3x+2)^2} + \frac{1}{(3x+2)^3} + \dots$ be a infinite geometric series and $px^2 + qx + r = 0$ be a quadratic equation in one variable, here p, q, r are the real numbers; $p \neq 0$

- a. Find the series when $x = 1$ and what is the common ratio of the obtain series? 2
- b. Impose a condition on 'x' under which the given series will have a sum up to infinity and find the sum. 4
- c. If 'a' and 'b' be the roots of this given equation, then show that $a + b = \frac{-q}{p}$ and $ab = \frac{r}{p}$ 4

3. ► $a = \log_p(qr)$, $b = \log_q(rp)$, $c = \log_r(pq)$ and $F(x) = x^3 + 6x^2 + 11x + 6$.

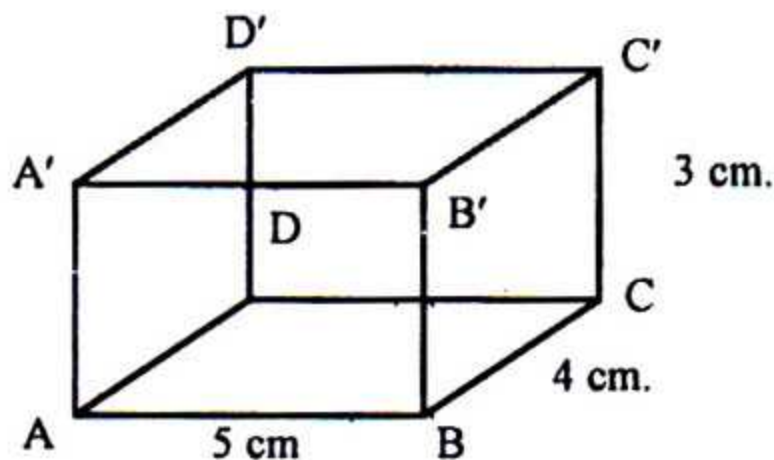
- a. When $C = 2$, then show that, $r = \sqrt{pq}$ 2

- b. If $F(x)$ yields the same remainder upon division by $x - u$ and $x - v$ where $u \neq v$, show that, $u^2 + v^2 + uv + 6u + 6v + 11 = 0$. 4
- c. Prove that, $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} = 1$. 4

Group B — Geometry and Vector

4. ► The perpendiculars AD, BE and CF from the vertices to the opposite sides of the acute angled triangle ABC meet at the point O.
- a. If $AC = 5$ cm, $CD = 3$ cm, then find the length of AD. 2
- b. Prove that, $AO \cdot OD = BO \cdot OE = CO \cdot OF$. 4
- c. Show that, $BC \cdot CD = AC \cdot CE$. 4
5. ► P (8, 3), Q (3, 8) and R (-2, 3) are the three vertices of a triangle 'S' and 'T' be the middle points of the sides PQ and PR respectively.
- a. Find the slope of QR. 2
- b. Show that, ΔPQR is an isosceles triangle and its area is 25 square units. 4
- c. Prove with the help of vectors that, $ST \parallel QR$ and $ST = \frac{1}{2} QR$. 4

6. ►



- a. Find the volume of the solid mentioned in the diagram. 2
- b. The radii of three metal solid sphere is AB, BC and CC' respectively. A new solid sphere is formed by melting the three spheres. Determine the radius and whole surface area of new sphere. 4
- c. A rectangle of size of ABCD surface of the solid is revolved about the greater side. Find the volume and the total surface area of the solid formed. 4

Group C — Trigonometry & Probability

7. ► $\cot\theta + \operatorname{cosec}\theta = m$.
- a. Find the value of $\operatorname{cosec}\theta - \cot\theta$. 2
- b. If $m = 2$, then show that,

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$$
 4
- c. If $m = \sqrt{3}$, then find the value of θ where $0 \leq \theta \leq 2\pi$ 4
8. ► A bag contains 10 red (R), 5 black (B), 8 white (W) and 6 yellow (Y) marbles. A marble is chosen at random from the bag.
- a. What is random experiment? 2
- b. What is probability that the marble will be yellow or black? 4
- c. Show that the probability that the marble will be red or white or black is equal to the probability that the marble will not be yellow. 4

[N.B — Answer all the questions. Each question carries one mark. Block fully, with a ball-point pen, the circle of the letter that stands for the correct/best answer in the "Answer Sheet" for Multiple Choice Questions Examination.]

1. $2^\circ = ?$

- (a) $\frac{p^c}{45}$ (b) $\frac{p^c}{90}$
 (c) $\frac{p^c}{180}$ (d) $\frac{p^c}{360}$

2. What is the volume of a pyramid in cubic units?

- (a) $\frac{1}{3} \times \text{area of the base} \times \text{height}$
 (b) $\frac{1}{2} \times \text{area of the base} \times \text{height}$
 (c) $\text{area of the base} \times \text{height}$
 (d) $\frac{3}{4} \times \text{area of the base} \times \text{height}$

3. If $\cos\theta = \frac{4}{5}$ and θ is acute angle, then $\operatorname{cosec}\theta = ?$

- (a) $\frac{3}{5}$ (b) $\frac{2}{5}$
 (c) $\frac{5}{3}$ (d) $\frac{5}{2}$

Answer to the questions no. 4 and 5 to the information given below:

In $\triangle ABC$, $AB = AC = 5\text{cm}$, $AD \perp BC$ and $BC = 6\text{cm}$.

4. Area of $\triangle ABC$ in sq. cm?

- (a) 12 (b) 13 (c) 14 (d) 15

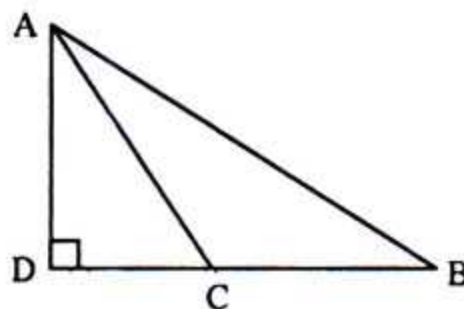
5. If the angle between AB and AD is θ , then $\tan\theta = ?$

- (a) $\frac{3}{4}$ (b) $\frac{2}{3}$
 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

6. If $7^x = y$, then which one is correct?

- (a) $x = 7\log y$ (b) $x = \log_7 y$
 (c) $x = \log_y 7$ (d) $x = \log_7 y$

In the light of the given figure, answer the questions no. 7 and 8:

7. Which is the orthogonal projection of AC on DB ?

- (a) AD (b) DC
 (c) DB (d) CB

8. $\angle B$ is an acute angle, then what is the value of AC^2 ?

- (a) $AB^2 + BC^2 - 2BC \cdot CD$
 (b) $AB^2 + BC^2 - 2BC \cdot BD$
 (c) $AB^2 + BC^2 + 2AC \cdot CD$
 (d) $AB^2 + BC^2 + 2AB \cdot AD$

9. Which is the value of one exterior angle of an equilateral triangle?

- (a) 120° (b) 180°
 (c) 270° (d) 360°

10. If $\sin 3A = \cos 3A$, then which one is the value of 'A'?

- (a) 15° (b) 20°
 (c) 30° (d) 40°

11. If $c(x + a) < b$ and $c > 0$, then which is correct?

- (a) $x < \frac{b}{c} + a$ (b) $x > \frac{b}{c} + a$
 (c) $x < \frac{b}{c} - a$ (d) $x > \frac{b}{c} - a$

12. An unbiased dice and two coins are thrown at a time. How many probability induced?

- (a) 24 (b) 12
 (c) 6 (d) $\frac{1}{12}$

13. Which is the co-efficient of 'x' in expansion of $(1 + 3x)^5$?

- (a) 1 (b) 5 (c) 10 (d) 15

14. \underline{a} , \underline{b} , \underline{c} are the position vectors of the points A, B, C respectively. If C divides AB internally in the ratio 5 : 11, then $\underline{c} = ?$

(a) $\frac{5b + 11a}{16}$ (b) $\frac{11b + 5a}{16}$
 (c) $\frac{5b - 11a}{16}$ (d) $\frac{11b - 5a}{16}$

15. If $\frac{x-5}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$, where A and B are rational number, then which is the value of A?

(a) -3 (b) -2
 (c) 1 (d) 2

16. Which is the sum of the series $0.12 + 0.0012 + 0.000012 + \dots$?

(a) $\frac{4}{3}$ (b) $\frac{4}{33}$ (c) $\frac{4}{333}$ (d) $\frac{4}{3333}$

17. If $f(x) = 3x + 1$, $0 \leq x \leq 2$, then what is the range of f ?

(a) $0 \leq y \leq 2$ (b) $1 \leq y \leq 2$
 (c) $0 \leq y \leq 7$ (d) $1 \leq y \leq 7$

18. If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, then which one is correct?

(a) $a + b = -ab$ (b) $a + b = ab$
 (c) $a + b = 1$ (d) $a + b = -1$

19. Any set A—

- i. will be an infinite set if and only if A is equivalent to its proper subset
 ii. if the number of the elements are n, then $n(P(A)) = 2^n$
 iii. is a subset of itself

Which one is correct?

(a) i and ii (b) i and iii
 (c) ii and iii (d) i, ii and iii

20. What is mean by 's' in $\sqrt{s(s-a)(s-b)(s-c)}$?

- (a) area of triangle
 (b) area of circle
 (c) half perimeter of triangle
 (d) half perimeter of circle

21. If $S = \{(1, 4), (2, 1), (3, 0), (4, 1), (5, 4)\}$, then—

- i. the range of the relation S is $\{4, 1, 0\}$
 ii. the inverse relation of S is $\{(4, 1), (1, 2), (0, 3), (1, 4), (4, 5)\}$
 iii. S is a function

Which one is correct?

- (a) i and ii (b) i and iii
 (c) ii and iii (d) i, ii and iii

22. When the roots of the equation $ax^2 + bx + c = 0$ will be real, unequal and rational?

- (a) discriminant is positive
 (b) discriminant is negative
 (c) discriminant is positive and perfect square
 (d) discriminant is negative and perfect square

Answer to the questions no. 23 and 24 to the information given below:

$$F(x) = \sqrt{x-1}$$

23. If $F(x) = 5$, then the value of 'x' = ?

(a) $\sqrt{10}$ (b) $\sqrt{26}$
 (c) 10 (d) 26

24. Which is true?

- (a) $\text{Dom } F = \{x \in \mathbb{R} : x \neq 1\}$
 (b) $\text{Dom } F = \{x \in \mathbb{R} : x \geq 1\}$
 (c) $\text{Dom } F = \{x \in \mathbb{R} : x \leq 1\}$
 (d) $\text{Dom } F = \{x \in \mathbb{R} : x < 1\}$

25. If $9^{2x} = 3^{x+1}$ then $x = ?$

(a) - (b) 0
 (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

Ans.	1	(b)	2	(a)	3	(c)	4	(a)	5	(a)	6	(d)	7	(b)	8	(b)	9	(a)	10	(a)	11	(c)	12	(a)	13	(d)	14	(a)	15	(d)
	16	(b)	17	(d)	18	(b)	19	(d)	20	(c)	21	(d)	22	(c)	23	(d)	24	(b)	25	(c)										